Abstracts for the MAA Undergraduate Poster Session

Baltimore, MD
January 18, 2019

MATHEMATICAL ASSOCIATION OF AMERICA

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Organized by

Eric Ruggieri
College of the Holy Cross

and

Chasen Smith
Georgia Southern University

Dear Students, Advisors, Judges and Colleagues,

If you look around today you will see approximately 350 posters and more than 550 student presenters, representing a wide array of mathematical topics and ideas. These posters showcase the vibrant research being conducted as part of summer programs and during the academic year at colleges and universities from across the United States and beyond. It is so rewarding to see this session, which offers such a great opportunity for interaction between students and professional mathematicians, continue to grow.

The judges you see here today are professional mathematicians from institutions around the world. They are advisors, colleagues, new PhDs, and administrators. Many of the judges signed up when they registered for the conference, but there are also a number of judges here today who volunteered on site. Their support is vital to the success of the session and we thank them.

We are supported financially by Tudor Investments and Two Sigma. We are also helped by the members of the MAA Committee on Undergraduate Students (CUS) in some way or other. They are: Emily Cilli-Turner, James Collins, Janine Janoski, Darci Kracht; Emille Davie Lawrence; Aihua Li; Sara Louise Malec; Rhonda McKee; Stacy Ann Muir; Andy Niedermaier; Pam Richardson; Peri Shereen; Hortensia Soto; and Violeta Vasilevska. There are many details of the poster session that begin with putting out the advertisement in FOCUS, making sure the online submission system works properly, and organizing poster boards and tables in the room we are in today that are attributed to Hortensia Soto (MAA Associate Secretary), Kenyatta Malloy (MAA), and Donna Salter (AMS).

Our online submission system and technical support is key to managing the ever-growing number of poster entries we receive. Thanks to MAA staff, especially Kenyatta Malloy, for her work managing the system this year. Preparation of the abstract book is a time-consuming task. Thanks to Beverly Ruedi for doing the final production work on the abstract book.

We would also like to thank James Collins (University of Mary Washington), Emille Davie Lawrence (University of San Francisco), Pam Richardson (Westminster College), Peri Shereen (California State University Monterey Bay), and Doug Ensley (MAA) for their help in authoring the new judging form, as well as Aihua Li (Montclair State University) and Peri Shereen for organizing an orientation for the judges.

Thanks to all the students, judges, volunteers, and sponsors. We hope you have a wonderful experience at this years poster session!

Eric Ruggieri
College of the Holy Cross

Chasen Smith
Georgia Southern University

Organized by the MAA

Committee on Undergraduate Students

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Their generosity on behalf of the 2019 Undergraduate Student Poster Session enables students to interact with peers and role models in the mathematical sciences during the largest mathematics meeting in the world.

Titles, Authors, Advisors and Abstracts

1. Noetherian Rings with Unusual Prime Ideal Structures.

Anya Michaelsen Williams College
Advisor(s): Susan Loepp, Williams College

For a ring R, the set of prime ideals of R, called the spectrum of R, is a partially ordered set with respect to inclusion. Given a partially ordered set X, M. Hochster showed exactly when X can be realized as the spectrum of a commutative ring. It is unknown, however, when a partially ordered set can be realized as the spectrum of a commutative *Noetherian* ring. In 2016, C. Colbert showed that there exists an uncountable Noetherian commutative ring with Krull dimension at least 2 and a countable spectrum. We extend this result in two ways. First, we consider a spectrum with a countable and uncountable branch and discuss progress toward constructing a Noetherian ring with this spectrum. Second, we construct a 2-dimensional uncountable excellent ring with a countable spectrum. We will outline both constructions as well as future work to extend these results. This research was done as part of the 2018 SMALL REU program at Williams College.

2. Transition Matrices for Young's Representations of S_n

Sam Armon Macalester College
Advisor(s): Tom Halverson, Macalester College

The irreducible representations of the symmetric group S_n are indexed by integer partitions $\lambda \vdash n$. The corresponding simple modules are denoted $\{S_n^\lambda \mid \lambda \vdash n\}$, and the dimension of S_n^λ equals the number f_λ of standard Young tableaux of shape λ. In the 1920s, A. Young defined two bases of S_n^λ — the natural and seminormal bases — by describing the action of $\sigma \in S_n$ on vectors indexed by standard Young tableaux of shape λ. We give a formula for the entries in the transition matrix between the seminormal and natural bases, answering an open question in the representation theory of the symmetric group. Our method is to use a graph Γ_λ, which has vertices labeled by the standard tableaux of shape λ and colored edges corresponding to adjacent transpositions in S_n. This graph is the Hasse diagram of weak Bruhat order on standard tableaux. The entries in the transition matrix are calculated using weights on walks on Γ_λ. We generalize our method to the wreath product group $S_n \wr \mathbb{Z}_r$ and the Iwahori-Hecke algebra $H_n(q)$ of S_n.

3. Putting the "k" in Curvature: k-Plane Constant Curvature Conditions

Maxine Calle Reed College
Advisor(s): Corey Dunn, California State University, San Bernardino

Differential geometry studies properties of manifolds using the tools of calculus and linear algebra, and we can characterize the local behavior of manifolds by studying representative model spaces. Studying curvature in this context allows us to generate representative numbers for model spaces, particularly by taking measurements that are basis-independent. This research generalizes two such curvature invariants known as constant sectional curvature and constant vector curvature. While both conditions are well understood considering 2-plane sectional curvature in 3-dimensional model spaces, little is known with regard to higher dimensions. We generalize these curvature conditions using k-plane scalar curvature in Riemannian model spaces of arbitrary finite dimension, and many results are generalizations of known aspects of 2-plane constant curvature conditions. By studying these k-plane curvature invariants, we can further characterize model spaces by generating representative numbers for various subspaces. This research was conducted as a part of the 2018 REU program at California State University, San Bernardino.

4. The Hilbert Series of the Polynomial Representation of the Rational Cherednik Algebra of type A_{n1} in Characteristic p for $p|n1$

Merrick Cai Kings Park High School
Advisor(s): Daniil Kalinov, MIT

We study the polynomial representation $L_{t,c}(\tau)$ of the rational Cherednik algebra $H_{t,c}(S_n, \mathfrak{h})$ of type A_{n-1} in an algebraically closed field of positive characteristic p where $p|n-1$. In the $t = 0$ case, for all $c \neq 0$ we give a complete description of the polynomials in the maximal proper graded submodule $\ker B$, the kernel of the contravariant form B, and subsequently find the Hilbert series of the irreducible quotient $L_{0,c}(\tau)$. In the $t = 1$ case, we give a complete

description of the polynomials in ker B when the characteristic $p = 2$ and c is transcendental over $\mathbb{F}_2$, and compute the Hilbert series of the irreducible quotient $L_{1,c}(\tau)$.

5. A Generalization of the Indefinite Spectral Theorem

Andrew Lavengood-Ryan California State University, San Bernardino
Advisor(s): Corey Dunn, California State University, San Bernardino

One of the most profound and important theorems in Linear Algebra is the Spectral Theorem. This theorem is well-known for positive definite inner product spaces, but is less developed in indefinite inner product spaces.We aim to develop a generalization of the Indefinite Spectral Theorem by examining Jordan Form matrices and their associated metric. We start with an operator that is self-adjoint with respect to some inner product on a real vector space. We then complexify and extend the inner product to be Hermitian; from this, we determine the "best basis" for this matrix, forcing the metric to be a direct sum of Standard Involuntary Permutation matrices. The theorem we're after is simply a corollary of this result. This research has had applications in Differential Geometry, but would undoubtedly have further applications in Linear Algebra, as well.

6. On Characteristics of Hyperfields Obtained as Quotients of Finite Fields

Hahn Lheem PROMYS
Dylan Liu PROMYS
Advisor(s): Matthew Baker, Georgia Tech

Let p be an odd prime. We show that there exists an explicit form for the characteristic of the hyperfield $\mathbb{F}_p/G$ where $|G| = 1, 2, 3, 4$. Finally, we prove a general form of the characteristic for hyperfields where $|G|$ is prime.

7. The voiced *Tonnetz* and the $\mathcal{J}$-group, with illustrations in Schubert's B♭ major Sonata

Patrick DeBonis University of New Mexico
Advisor(s): Thomas Fiore, University of Michigan-Dearborn

Motivated by Schubert's Piano Sonata in B♭ Major, D. 960, we expand knowledge of the $\mathcal{J}$ group developed by Fiore and Noll. In the spirit of David Lewin, we use the PLR-group to make both global and local maps of the sonata, following Richard Cohn. We use the Structure Theorem of Fiore-Noll to find $\mathcal{J}$ group operations that realize some of these musical motions while preserving voice ordering. As an enrichment of the neo-Riemannian *Tonnetz* we develop a voice leading *Tonnetz* for the $\mathcal{J}$ group as a simplicial set, rather than simplicial complex. As we explore the topological structure of our *Tonnetz*, we observe the elements of the extended $\mathcal{J}$ group that preserve Cohn's hexatonic set. Finally, we propose three new groups of singular (!) matrices that accomplish major to diminished triad movement, motivated by Schubert's use of diminished triads. Main results: The geometric realization of the voice leading *Tonnetz* is a 6-fold cover of the neo-Riemannian *Tonnetz* and there does not exist a matrix that sends diminished chords to major chords compatible with transposition. UM-Dearborn REU

8. Virtual Complete Intersections in $\mathbb{P}^1 \times \mathbb{P}^1$

Amal Mattoo Harvard College
Jiyang Gao Massachusetts Institute of Technology
Yutong Li Haverford College
Advisor(s): Christine Berkesch, University of Minnesota Twin Cities

We study homological invariants of a finite set of points in $\mathbb{P}_1 \times \mathbb{P}_1$. We give examples where the size of a minimal free resolution for the vanishing ideal of the points depends on interactions between the cross ratios of the points in each copy of $\mathbb{P}_1$. We then examine virtual resolutions in the sense of Berkesch, Erman and Smith, which for points in $\mathbb{P}_1 \times \mathbb{P}_1$ are complexes that resolve the vanishing ideal only up to saturation by the irrelevant ideal. We use tools including Generalized Bézout's Theorem, degree analysis, and number-theoretic techniques, to give various sufficient conditions and necessary conditions for a virtual complete intersection. In particular, we completely classify the case where the points form a Ferrers diagram on a ruling of $\mathbb{P}_1 \times \mathbb{P}_1$. This research was carried out as part of the 2018 Combinatorics REU at the School of Mathematics at University of Minnesota, Twin Cities. We are grateful for the support of NSF RTG grant DMS-1745638.

9. Centralizer-like Subgroups Associated with the *n*-Engel Word Inside of Direct Product Groups

Maggie Reardon Reardon University of Wisconsin-Eau Claire
Advisor(s): Dandrielle Lewis, University of Wisconsin-Eau Claire

The primary goal of our project was to characterize centralizer-like subgroups associated with n-Engel words inside of a direct product of two groups. This idea originated from reading the paper by Kappe and Ratchford titled On Centralizer-like Subgroups Associated with the n-Engel Word. We proved conjectures about centralizer like subgroups associated with the 1-Engel word of a direct product, and we proved that the right n-Engel words of a direct product of two groups is equal to the direct product of the right n-Engel groups of each direct factor. These results provide insight into how we can further characterize centralizer-like subgroups in a direct product of groups. This research was conducted with Bridget Lee during an REU program at the University of Wisconsin-Eau Claire this past summer.

10. Characterization of Zigzag Inverse Semigroups

Jennifer Gensler California State University, Long Beach
Ronen Wdowinski Rice University
Advisor(s): David Milan, University of Texas at Tyler

Motivated by examples of C^*-algebras generated by semigroups, we investigate constructions of inverse semigroups from left cancellative categories such as the inverse semigroup of the path category of a directed graph. We give axioms that characterize inverse semigroups that are isomorphic to the set of zigzag maps on a left cancellative category. This work was completed as part of the REU program at University of Texas at Tyler.

11. Straightening Identities for Twisted Multiloop Algebras

Daniel Davidson Park University
Montana Miller Park University
Advisor(s): Samuel Chamberlin, Park University

Let $\mathfrak{g}$ be a complex simple Lie algebra. We examine the vector space formed by tensoring $\mathfrak{g}$ with the set of multi-variable Laurent polynomials. This vector space forms a Lie algebra. We then define a function σ that acts on this Lie algebra and consider the set of all elements fixed by σ. We define this set to be the twisted multi-loop algebra of $\mathfrak{g}$, denoted $T_m(\mathfrak{g})$. We have formulated and proved straightening identities for some elements of $T_m(\mathfrak{g})$. These identities can be used to define integral forms for these Lie algebras, which in turn could be used to investigate the representation theory of these algebras in positive characteristic.

12. Decomposing Permutations from Young Tableaux

Christopher Koch Butler University
Advisor(s): Amber Russell, Butler University

A Young Tableau is an arrangement of a finite number of boxes where each box contains a number. Utilizing two types of Tableau labelings, we can create a permutation that tracks how the entries change between the labelings. Young Tableaux have applications in Lie Theory and the main goal of this project is to use them to better understand a specific connection between different descriptions of the same Lie Theory object. In particular, we have related the permutations found from the Tableaux to the longest element of a Lie theoretic Weyl group, which are found using Dynkin diagrams. Both the Young Tableaux and Weyl group permutations appear in descriptions of irreducible components of Springer fibers. The ultimate goal of this project is to understand how these descriptions are connected using just the combinatorics. We currently have partial results towards this.

13. The Commuting Graph of Semi-direct Products of Cyclic Groups

Maria Diaz California State University, Fresno
Yuliana Segura California State University, Fresno
Advisor(s): Oscar Vega, California State University, Fresno

The commuting graph of a group G, denoted $C(G)$, is a simple graph whose vertex-set is $G - Z(G)$, and two vertices are connected if and only if they commute. Not much is known about this object, thus the underlying objective of this project is to increase the knowledge, and understanding, about commuting graphs of groups. Our project focuses on studying the commuting graph of certain families of groups. However, we are currently focusing our efforts on groups that are semi-direct products of cyclic groups.

14. Constructions and Deformations of $\mathbb{Z}_2$-graded $3|2$-dimensional Complex Associative Algebras

Ellie Lochner University of Wisconsin - Eau Claire
Jack Lazowski University of Wisconsin - Eau Claire
Advisor(s): Michael Penkava, University of Wisconsin - Eau Claire

$\mathbb{Z}_2$-graded complex associative algebras arise both in physics and mathematics. Here we look at the $3|2$-dimensional algebras, and provide a construction of the moduli space, or the space of all such algebras up to isomorphism, using extensions of algebras of a lower dimension. We can use our knowledge of these lower dimensional algebras to construct the higher dimensional ones. Another interesting aspect of this work is our ability to study the deformations of these algebras. We analyze these deformations of algebras via computation of versal deformations.

15. Numerical Range of Toeplitz Matrices over Finite Fields

Maddi Guillaume Taylor University
Amish Mishra Taylor University
Advisor(s): Derek Thompson, Taylor University

This work characterizes the kth numerical range of all $n \times n$ Toeplitz matrices with a constant main diagonal and another single, nonzero diagonal, where the matrices are over the field $\mathbb{Z}_p[i]$, with p a prime congruent to $3 \mod 4$. For $k \in \mathbb{Z}_p^*$, the kth numerical range is always equal to $\mathbb{Z}_p[i]$ with the exception of the scaled identity. Similar techniques are used to discover a general connection between the 0th numerical range and the kth numerical range. Lastly, a conjecture is given regarding the general numerical range of all triangular Toeplitz matrices.

16. Solutions to Matrix Equation $X^2AX = AXA$ Over Finite Fields of Prime Order

Saroj Niraula Caldwell University
Advisor(s): Aihua Li, Montclair State University

In papers by Li, Randall, and Mosteig, complete solutions to the matrix equation $X^2AX = AXA$ $(*)$ are established when A is a 2 by 2 matrix over complex field. Certain solutions to higher dimensional cases are also provided. In real or complex number systems, problem of solving equation $(*)$ can be reduced to the case when matrix A is in its Jordan normal form. The equation $(*)$ has not been studied over finite fields of prime order. In current research, we want to find solutions to $(*)$ over such fields. We first study the case when A is a 2 by 2 matrix in Jordan normal form over a finite field of prime order. We solve equation $(*)$ and analyze solutions based on their singularity. Unlike in the real or complex number systems, some 2 by 2 matrices in a finite field of prime order are not similar to any Jordan normal matrix. We try to categorize these differing matrices, and investigate corresponding solutions to $(*)$. We provide complete solutions to equations $(*)$ over fields of order 2 and 3. For finite fields of order greater than 3, we give certain solutions and categorize solution types.

17. Coadjoint orbits for the group of unipotent upper triangular matrices.

Caleb Lyon Houghton College
Nathaniel Parks Houghton College
Advisor(s): Brandon Bate, Houghton College

The Lie group G_n of unipotent upper triangular $n \times n$ matrices acts on its Lie algebra $\mathfrak{g}_n$ via the adjoint representation. From this, one defines the coadjoint representation of G_n on the dual space $\mathfrak{g}_n^*$. Although Kirillov's orbit method has long established a correspondence between the space of coadjoint orbits and equivalence classes of irreducible unitary representations of G_n, a general classification of coadjoint orbits of G_n remains an open problem. In this research, we present an algorithm for classifying these orbits along with accompanying computational results for small n.

18. The splitting criterion in the hyperoctahedral group and other results on conjugacy classes

McKenzie Scanlan University of Wisconsin-Eau Claire
Advisor(s): aBa Mbirika, University of Wisconsin-Eau Claire

We examine elements in both the hyperoctahedral group $\mathbb{G}(2,1,n)$ and an analogue of the alternating subgroup $A_n \subseteq S_n$ in $\mathbb{G}(2,1,n)$—namely, the orientation-preserving symmetries, which we denote $\mathbb{A}(2,1,n)$. In particular, we investigate their corresponding conjugacy classes and centralizers. We employ known symmetric group results

on both the size of a conjugacy class and the size of a centralizer for a particular element $\sigma \in S_n$ to determine an analogous result in the hyperoctahedral group setting. We study the previously known splitting criterion for conjugacy classes in the alternating subgroup of the symmetric group and we generalize these results to find the analogue splitting criterion in the hyperoctahedral group.

19. Transplanting Trees: Chromatic Symmetric Function Results through the Group Algebra of S_n

Joshua Kazdan Stanford
Sofia Martinez University of California Riverside
Advisor(s): Angele Hamel, Laurier University

One of the major outstanding conjectures in the study of chromatic symmetric functions states that trees are uniquely determined by their chromatic symmetric functions. Though verified on graphs of order not exceeding twenty-nine, this result has been proved only for the subclass of trees known as spiders. Furthermore, Martin et al. have shown, using combinatorial methods, that the chromatic symmetric function can recover the degree sequence and path sequence of a given graph. In this paper, using the definition of the chromatic symmetric function that emerges through the S_n group algebra, we reconstruct many known results about trees from their chromatic symmetric functions. Additionally, we prove a weaker result about the uniqueness of trees through a related function in the group ring of S_n.

20. Developing Straightening Identities in MATLAB for the Onsager Algebra of $\mathfrak{sl}_2$

Denver Strong William Jewell College
Advisor(s): Azadeh Rafizadeh, William Jewell College

We have developed equations in order to formulate straightening identities in a subalgebra of the Loop algebra of the special linear lie algebra ($\mathfrak{sl}_2$). This particular subalgebra is known as the Onsager Algebra, denoted $\mathcal{O}$. The importance of the Onsager Algebra is seen due to its similarities to the tetrahedron algebra and Dolan Grady relations used heavily in quantum physics. Since the Onsager Algebra is matrix based, we use a preferred order of $x_m x_n w_l$ where $m, n \in \mathbb{Z}$, $m > n$, and $l \in \mathbb{N}$. We have produced equations with the help of Mathematica and MATLAB.

21. High-Dimensional Sphere Packings

Zachary Stier Princeton University
Advisor(s): Alex Kontorovich, Rutgers University

The Apollonian circle packing, generated from four mutually-tangent circles in the plane, has inspired over the past half-century the study of other classes of space-filling packings, both in two and in higher dimensions. More recently, this has yielded a connection between n-dimensional packings and configurations of planes in $\mathbb{H}^{n+1}$ for various quadratic forms in $n+2$ variables. In particular, Vinberg's algorithm, in conjunction with Kontorovich and Nakamura's Structure Theorem, allows us to ask questions about whether certain Coxeter diagrams in $\mathbb{H}^{n+1}$ for a given quadratic form admit a packing at all. Further, Kontorovich and Nakamura's Finiteness Theorem shows that there only exist finitely many classes of these packings, none of which are in dimension 21 or above. In this work, we systematically determine and enumerate all known examples of higher-dimensional sphere packings arising in this way. This research was performed at the DIMACS REU at Rutgers Univeristy-New Brunswick.

22. Combinatorial Neural Codes: A Study of k-Inductively Pierced Codes and Toric Ideals

Melissa Beer Franklin College
Alexandra Newlon Colgate University
Kira Laws Appalachian State University
Thomas Elgin University of South Carolina
Matthew Hertel Michigan State University
Advisor(s): Robert Davis, Harvey Mudd College

Combinatorial neural codes are sets of 0/1 vectors that encode when a fixed set of neurons are firing. Place cells are certain neurons whose firing patterns are determined by an animal's location within an arena. One area of interest is finding Venn diagram-like figures, called Euler diagrams, that correspond to given combinatorial neural codes for collections of place cells. Gross, Obatake, and Youngs have recently used algebraic techniques to study combinatorial neural codes, with specific interest on their toric ideals. In this poster session, we will demonstrate our progress in analyzing these codes, with specific interest in the Grbner bases of toric ideals. This includes studying the effects of using different weighted monomial orderings, identifying the indispensable binomials of external Euler diagrams, and exploring the toric ideals of 1-inductively pierced diagrams. The research presented was conducted at the 2018 SURIEM Program at Michigan State University.

23. (2,3,7)-Nielsen classes of the Alternating Groups

Vincent Noh Grinnell College
George Ge Grinnell College
Advisor(s): Jennifer Paulhus, Grinnell College

Over the course of this project we investigated the $(2,3,7)$-*Nielsen Classes* of the Alternating Groups (A_n). These are triples of elements in each A_n, x, y, z such that $z = (xy)^{-1}$, $x^2 = y^3 = z^7 = e$ and $\{x, y, z\}$ generates A_n. This problem is motivated by others in Topology and Algebra, especially the decomposition of Jacobian varieties. Using results by Marston Conder and John McKay, we found at least one example of x, y, z in all A_n that are Hurwitz groups (groups with (2,3,7)-Nielsen classes) except for eight cases (A_{63}, A_{72}, A_{93}, A_{100}, A_{108}, A_{114}, A_{121}, and A_{128}). Additionally, we greatly simplified an inner product formula for decomposing Jacobian varieties that correspond to Hurwitz groups with automorphism group A_n and we showed that this decomposition is never simple (ie they are further decomposable).

24. Curves, Conics and Cryptography

Ryan Zesch California Polytechnic State University, San Luis Obispo
Joel Pion California Polytechnic State University, San Luis Obispo
Advisor(s): Eric Brussel, California Polytechnic State University, San Luis Obispo

Elliptic curves are a deep area of mathematics with many surprising applications to real life. Pell conics are the less studied little brother of elliptic curves. Our work focused on categorizing the group structure of Pell conics over various rings. Additionally, we have taken some well known elliptic curve algorithms and converted them to work on Pell conics. We also investigated the map from $\mathbb{Z}$-points to $\mathbb{F}_p$-points of certain Pell conics, and found that it is sometimes but not always surjective.

25. Rotational Row-Complete Latin Squares—from Quilts to Sequenceable Groups

Zhaopeng Li Colorado College
Jerrell Cockerham Colorado College
Advisor(s): Beth Malmskog, Colorado College

Cockerham and Lis research project explore these questions: for what n is it possible to create rotational row complete Latin squares—squares where each row follows the same pattern of transitions, though the pattern begins in a different place each time. Many such patterns have been found over years, but it is not known whether all have been discovered. Cockerham and Lis research began with compiling all known results. They then did a computational check to determine all patterns for small to moderate size n. Based on their findings, they attempted to prove that new patterns work for some infinite class of sizes, or that no additional new patterns can exist.

26. Matrix Square Roots of Polynonials

Trung Vu St. Olaf College
Advisor(s): Kosmas Diveris, St. Olaf College

In this research, we consider matrix factorizations of a polynomial where the two matrices appearing in the factorization are the same, which we call "matrix square roots." The main result is that any polynomial in $\mathbb{R}[x_1, \ldots, x_n]$ admits a matrix square root. Our proof is constructive and provides an algorithm for constructing these matrices.

27. Variations of Gershgorin Theorem

Courtney Hauf SUNY Brockport
Advisor(s): Gabriel Prajitura, SUNY Brockport

One of the reasons Gershgorin Theorem cannot be extended to infinite dimensional Hilbert spaces is the fact that the radii involved may be infinite. We explore ways of replacing the usual radii (which are in l^1) with some Euclidean (l^2) version.

28. Hilbert Series of Quasi-invariant Polynomials

Archer Wang MIT PRIMES
Advisor(s): Xiaomeng Xu, MIT

The space of quasiinvariant polynomials of order m generalizes that of symmetric polynomials: under the action of the symmetric group, the polynomials remain invariant to a certain order. Their Hilbert series were studied by Felder and Veselov in fields of characteristic 0. In this paper, we investigate the Hilbert series of quasiinvariant polynomials that are divisible by a generic homogeneous polynomial. In addition, we continue the work of previous results regarding their Hilbert series in fields of prime characteristic.

29. Cohomology Groups of the dual Steenrod Algebra

Ryan Kim MIT PRIMES-USA Program/Thomas Jefferson High School for Science and Technology
Advisor(s): Sanath Devalapurkar, MIT

In homological algebra, the cohomology groups of a group action on a module are one of the most powerful structures available for understanding the action. We study the modulo 2 dual Steenrod algebra $\mathcal{A}_*$, a structure in homological algebra formed by taking the dual of the Steenrod algebra for $p = 2$. Milnor previously showed that the dual Steenrod algebra is a polynomial ring over $\mathbb{Z}/2$, and we investigate the cohomology groups of the canonical conjugation action χ of $\mathbb{Z}/2$ on $\mathcal{A}_*$. The first step of such a computation is computing $\mathcal{A}_*^\chi$, the fixed points subalgebra of $\mathcal{A}_*$ under χ. We introduce some important elements of $\mathcal{A}_*^\chi$, and we then study some asymptotics and bounds on the dimensions of $\mathcal{A}_*$ of $\mathcal{A}_*^\chi$. In doing so, we study the Diophantine equation $\sum_{i=1}^{\infty} a_i(2^i - 1) = n$ and its asymptotic properties.

30. Results on Leibniz n-algebras from the Category $\mathbb{U}_n(\mathfrak{L}_b)$

Min Soo Kim Vanderbilt University
Advisor(s): Marcelo Disconzi, Vanderbilt University

We study the Leibniz n-algebra $\mathbb{U}_n(\mathfrak{L})$, whose multiplication is defined via the bracket of a Leibniz algebra $\mathfrak{L}$ as $[x_1, \dots, x_n] = [x_1, [\dots, [x_{n-2}, [x_{n-1}, x_n]] \dots]]$. We the simplicity of $\mathbb{U}_n(\mathfrak{L})$ when $\mathfrak{L}$ corresponds to a simple Lie algebra. An analogue of Levi's theorem for Leibniz algebras in $\mathbb{U}_n(\mathfrak{L}_b)$ is established and it is proven that the Leibniz n-kernel of $\mathbb{U}_n(\mathfrak{L})$ for any semisimple Leibniz algebra $\mathfrak{L}$ is the n-algebra $\mathbb{U}_n(\mathfrak{L})$.

31. Context Directed Sorting: Robustness and Complexity

Leigh Foster Metropolitan State University of Denver
Manaswinee Bezbaruah University of Minnesota Twin Cities
Henry Fessler Montana State University
George Spahn Brown University
Advisor(s): Marion Scheepers, Boise State University

Ciliates are single-cell organisms with two nuclei. The DNA sorting processes that naturally occur in ciliates are analogous to two particular operations on permutations. These sorting operations are of combinatorial interest outside of biology. The operations are efficient but not every permutation is sortable. We look at the sortability of permutations through the lens of graph theory and matrices. If a permutations is sortable, we show that any possible sequence of these operations will sort it. We give a complete characterization of sortable permutations, and use it to produce an efficient algorithm for determining sortability. In the unsortable case, this algorithm gives the possible termination states that can result from these sorting operations. In the cases where sorting doesn't work, we can represent the interactions between permutations from a game theoretic point of view. We examine one- and two-player games based on these permutations, and develop techniques for determining a winning strategy.

32. Sandpile Groups of Cayley Graphs of $\mathbb{F}_2^r$

Jiyang Gao MIT
Jared Marx-Kuo University of Chicago
Vaughan McDonald Harvard University
Advisor(s): Victor Reiner, University of Minnesota

The **sandpile group** is a subtle isomorphism invariant associated to a graph. Past work has focused on the sandpile group of the n-dimensional hypercube. In this project, we perform a more general analysis on the Cayley graph of the group $\mathbb{F}_2^r$ with respect to any generating set. While the p-Sylow component of the sandpile group has been classified for $p \neq 2$, significantly less is known about the 2-Sylow component. In this paper, we use ring theory to prove a

sharp upper bound for the largest 2-Sylow subgroup in the sandpile group of an arbitrary Cayley graph. In the case of the hypercube, we use this bound to determine the n-largest cyclic factors. Additionally, we determine the number of 2-Sylow invariant factors in a majority of the cases. We also partially classify the number of 2-Sylow subgroups in the sandpile group and make further reductions into determining its structure. Using these reductions, we provide a full classification of the sandpile group for the $r = 2$ case and other enlightening results for small r cases. This research was conducted at the 2018 Combinatorics REU at the University of Minnesota, Twin Cities.

33. Feynman Operational Calculus

Lorenzo Riva Creighton University
Advisor(s): Lance Nielsen, Creighton University

The forthcoming paper "*Combining continuous and discrete phenomena for Feynman's operational calculus in the presence of a* (C_0) *semigroup and Feynman-Kac formulas with Lebesgue-Stieltjes measures*" (by L. Nielsen, to appear in *Integral Equations and Operator Theory*) contains, as its main result, an evolution equation which serves to describe how Feynman's operational calculus evolves with time in the presence of a (C_0) semigroup of linear operators. There are several examples in this paper which give rise to so-called, Feynman-Kac formulas with Lebesgue- Stieltjes measures (first investigated from a function space integral point of view by M. L. Lapidus). However, due to the different approach, the Feynman-Kac formulas obtained in the paper by Nielsen differ from those obtained by Lapidus. An associated operator differential equation (essentially a nonhomogeneous Schrodinger's equation) is also obtained in the paper. This poster will explain the newly-found Feynman-Kac formulas for the free Hamiltonian operator on $L^2(\mathbb{R}^\ltimes)$ and some results obtained by using different generators, like the square root of the Laplacian and various translation generators.

34. 2-Filtrations of Recurrently Generated Polynomials

Julie Campos University of New Mexico
Kapil Chandran Princeton University
Young Han Kim Stanford University
Advisor(s): Khang Tran, CSU Fresno

The n-filtration of a polynomial is the new polynomial obtained by keeping only those terms with exponent divisible by n. We apply 2-filtration to a recursively generated sequence of complex polynomials $P_m(z)$ satisfying a generating function $\sum_{m=0}^{\infty} P_m(z)t^m = \left(1 + A(z)t + B(z)t^2\right)^{-1}$, where $A(z)$ and $B(z)$ are linear polynomials. We identify sufficient conditions on the coefficients of $A(z)$ and $B(z)$ so that the 2-filtration of each $P_m(z)$ has only real and purely imaginary zeros. We also conjecture natural generalizations for n-filtrations. This research was conducted at the CSU Fresno REU in the summer of 2018.

35. Kantorovich Duality and Optimal Transport Problems on Magnetic Graphs

Sawyer Robertson University of Oklahoma
Advisor(s): Javier Alejandro Chvez-Domnguez, University of Oklahoma

Working first in the setting of finite combinatorial graphs, we explore Lipschitz function spaces, and give constructive results concerning norm-preserving extensions from certain subgraphs, as well as the identification of convex extreme points in the unit ball. We then move over to so-called magnetic graphs, which are equipped with a discrete analogue of a magnetic vector potential known as a signature. In this setting, we establish a Kantorovich-type duality result for 'signed' Lipschitz spaces, as well as the identification of the convex extreme points of the unit ball in this space. Finally, we apply this theory with the help of magnetic lift graphs to give some semi-constructive results concerning 'magnetic' optimal transport problems formulated on these graphs.

36. An Application of Abel's Method to the Inverse Radon Transform

Alexander Nolte Tufts University
Julie Sherman University of Minnesota - Twin Cities
Joseph David
Advisor(s): **Zair Ibragimov, California State University, Fullerton**

We investigate a method of approximating the inverse Radon transform on the plane by integrating against a family of smooth kernels. For peicewise smooth functions $f \in L^1(\mathbb{R})^2$, we establish that our method of approximation

converges pointwise to a local average of f and rule out Gibbs Phenomena. We discuss geometric properties of the kernel and their implications for computer implementation. We suggest some applications and present an example. This work was done with the IRES Program in Uzbekistan.

37. Conditions for Lipschitz Continuity on Post-Critically Finite Self-Similar Sets

Anchala Krishnan University of Washington Bothell
Benjamin York Bowdoin College
Advisor(s): Luke Rogers, University of Connecticut

Kigami's theory of analysis on post-critically finite self-similar (pcfss) sets applies to many well-known fractals, such as the Sierpinski Gasket. In this theory, functions with Laplacian in L^1 are Lipschitz in an intrinsic metric called the resistance metric. However, there are other useful metrics on these sets: One fundamental example is the geodesic metric on the Sierpinski gasket in harmonic coordinates. One may recognize this fractal as a one-dimensional C^1 subset of $\mathbb{R}^2$ which carries a measurable Riemannian structure. By analyzing the self-similar structure of the Green's operator which inverts the Laplacian, we give sufficient conditions for the Lipschitz and Hölder continuity of functions with L^p Laplacian on pcfss sets endowed with measures and metrics from a general class that includes the Riemannian structure on the harmonic Sierpinski gasket.

38. Matrix Models for the Circular Beta Ensemble and an Exploration of Its Limiting Behavior

Eli Cytrynbaum Williams College
Advisor(s): Mihai Stoiciu, Williams College

The Circular Beta Ensemble is a class of matrices whose eigenvalues model the behavior of a Coulomb gas at inverse temperature beta. By constructing specific matrix models for this ensemble, we are able to investigate the limiting behavior of the point processes described by its eigenvalues in the extremes of temperature.

39. The Magnetic Spectrum on the Sierpinski Gasket

Ruoyu Guo Colgate University
Advisor(s): Joe Chen, Colgate University

The magnetic Laplacian on a connected graph is a Hermitian matrix with 1's on the diagonal, and suitably weighted complex numbers in the xy-entry if x and y are connected by an edge, and 0 if x and y are not connected. In this poster session, I will describe how to compute the eigenvalues of the magnetic Laplacian on the infinite Sierpinski Gasket (SG) graph, which is a famous self-similar fractal. By computing the Schur complement of the magnetic Laplacian from the $(n+1)$th-level graph approximation to the nth-level graph approximation, we can find a function $R(\lambda)$ which allows us to obtain the eigenvalues iteratively. The spectrum exhibits self-similarity and resembles a "butterfly."

40. Harmonic Mappings from Generalizations of Hypocycloids

Lauren Stierman Colorado College
Advisor(s): Jane McDougall, Colorado College

A family of generalized hypocycloids called rosettes display unique mathematical properties. The rosette is a harmonic mapping with canonical decomposition identical to the simple polynomials that generate the harmonic hypocycloid, but with additional factors that are hypergeometric 2F1 series. While the hypocycloid is well-known in the mathematical realm, the rosette was only discovered in 2016. This project explores the intricacies of rosettes and their relationship with hypocycloids, through an analysis of the relationships between relevant formulae and the use of the computer algebra system Mathematica. The graphics of hypocycloids and rosettes that this system generates are of particular significance to this research endeavor. Important findings include the following properties of the rosette: its nested behavior, the mirror image property on the boundary of its mappings, the equal and opposite tangent vectors that produce this property, and the distinct shapes that are generated based on the relative angle rotation of the functions of the canonical decomposition.

41. Linear Factorization of Hypercyclic Functions for Differential Operators

Jakob Hofstad St. Olaf College
Advisor(s): David Walmsley, St. Olaf College

Take some infinitely differentiable function and repeatedly differentiate it. The successive derivatives of this function form a sequence. What kinds of sequences appear? In some cases the sequence is periodic, or terminates at 0, but in most other cases the sequence is not only non-periodic, but actually dense! In this poster presentation, we'll show how to construct such a *hypercyclic function* for the derivative operator on the space of complex differentiable functions. We will also display our recent work on constructing such a hypercyclic function for any given differential operator as an infinite product.

42. Spectra of Kohn Laplacians on Spheres

John Ahn Bowdoin College
Emilee Cardin College of William & Mary
Mohit Bansil Michigan State University
Garrett Brown Harvard University
Advisor(s): Yunus Zeytuncu, University of MichiganDearborn

A CR-manifold is a submanifold in $\mathbb{C}^M$ with extra structure stipulating that the dimension of the complex part of its tangent space is pointwise invariant under some complex structure map. The Kohn Laplacian $\Box_b$ is a second order differential operator intrinsically defined on any CR-manifold. We study the spectrum of $\Box_b$ on unit spheres in $\mathbb{C}^N$ and revisit Folland's classical eigenvalue computation. Folland computes the eigenvalues of $\Box_b$ on $L^2(\mathbb{S}^N)$ using unitary representations; we instead use spherical harmonics. This approach enables us to write SymPy code that computes the eigenvalues of $\Box_b$ and similar second order differential operators defined on $L^2(\mathbb{S}^N)$. We also look at the growth rate of the eigenvalue counting function in this context. We also consider the growth rate of the eigenvalues of the perturbed Kohn Laplacian on the Rossi sphere in $\mathbb{C}^2$. We provide sharp upper and lower bounds on the maximum eigenvalues. This work was done at the NSF REU Site at the University of Michigan-Dearborn.

43. Eliminating Bias in Hong Kong Air Ventilation Assessments

Owen Levin University of Wisconsin
Advisor(s): David Dy, Hong Kong University of Science and Technology

After the outbreak of SARS in 2003, the Hong Kong government became concerned with poor air quality affecting the health of the city's inhabitants. Now, whenever a new building development is planned in Hong Kong, its impact on the air ventilation of the city is assessed through simulations and eventually wind tunnel experiments. If the building development would harm the air quality of the city, then it may not be built. However, current guidelines require picking a set of test points in the city and aggregating wind data from that set. The result of the assessment is sensitive to exactly where test points are placed. Through either accidental bias or careful adversarial placement, buildings harming the city air ventilation may wrongly pass the assessment. We propose a norm on the set of possible sets of test points whose value corresponds to how biased a set of test points' result will be. Optimizing with respect to this norm allows us to choose test points for the assessment guaranteeing an unbiased result. This work was carried out at RIPS - Hong Kong 2018

44. Acoustic Classification of Bird Species Using Wavelets and Learning Algorithms

Song Yang Emporia State University
Advisor(s): Qiang Shi, Emporia State University

This poster presents our work on deriving an effective and efficient mathematical algorithm to identify bird species from bird calls. Classifying bird species based on their calls can be useful in real applications, such as determining the health of an ecosystem, or identifying hazardous species of birds near airports and reducing the bird-aircraft strikes. Having well-trained ornithologists to identify the characteristics of birds requires many man-hours, and the results may be subjective. This research is intended to develop a semi-automatic classification algorithm. We first performed a wavelet decomposition algorithm over more than 1200 syllables from 12 different bird species, and then extracted a set of 8 parameters from each instance. The dataset formed by the instances and associated parameters was used to train and test different classifiers. Our results showed that among all the classifiers we tested, Quadratic Support Vector Machines and Random Forest achieved the highest classification rate.

45. Wiener-Hopf Integral Equation Model: Underwater Applications

Cole Foster Roger Williams University
Advisor(s): Yajni Warnapala, Roger Williams University

The objective of this research project is to find a numerical solution to the Neumann boundary value problem for the Wiener-Hopf Integral Equation Model. The Wiener-Hopf Integral Equation has applications in radiative transfer, electromagnetics, and optical oceanography. The mathematical model of this integral equation will yield convergence results of incoming external waves over the surface of spherical shapes that satisfy Greens Theorem. The modeling of these shapes will be done using the Galerkin method, a type of numerical approximation using the Gaussian Quadrature nodes. Inspired by the loss of Malaysian Airline flight 370, this study into scattering theory and optical oceanography would help provide the theoretical framework for understanding light propagation in the ocean, which can aid in the search of submerged objects.

46. Modeling the Spread of Radioactive Contaminant in Fractured Aquifers Surrounded by Porous Rock

Eric Montoya CSU Chico
Susan Ye Brown University
Advisor(s): Sergei Fomin, CSU Chico

Our research models the spread of radioactive contaminant by mass transfer through fractures and aquifers in subsurface porous rock. This is a topic of consideration for nuclear waste repository design, wherein radioactive waste must be isolated from the environment and groundwater sources. By solving a system of fractional differential equations for the steady-state case, with initial state conditions that model the radioactive contaminant and environmental conditions, we can predict the zones of contamination. The model accounts for the physical processes of diffusion, dispersion, and advection due to fluid flow, in addition to intrinsic properties of the system, such as rock porosity, radioactive half-life of the contaminant, and anomalous fracture patterns in the rock. Due to prior experimental and theoretical results, this work utilizes these fractional-order terms to model the mass transport in the aquifer and porous rocks. The solutions can then be used to examine contaminant zones in various real-world conditions to help outline safety procedures and guidelines for disaster prevention. Our work was done in an NSF funded REU program at California State University, Chico.

47. Semi-supervised Learning for Visual Semantics

Caitlin Shener University of California Los Angeles
Advisor(s): Bhaven Mistry, University of California Los Angeles

The interpretation of visual semantics from camera signals is an important part of self-driving cars and smart traffic systems. However, due to limited supplies of labeled traffic image data, it is very difficult to run traditional deep learning algorithms to solve tasks like identifying lane markings and tires. In this project we will explore several deep learning object detection algorithms, computational geometry tactics, and adversarial approaches to solve semi-supervised learning problems concerning traffic data. Namely, we aim to localize tires and detect traffic lanes in a set of images and generate more training data sets to improve the performance of the object detection. This research was completed at the Institute for Pure & Applied Mathematics' program Research in Industrial Projects for Students – Hong Kong.

48. On the rank of matrix multiplication tensors of medium size

Jiahan Du University of California, Berkeley
Advisor(s): Olga Holtz, University of California, Berkeley

The multiplicative complexity of matrix multiplication for n by n matrices is an important topic in theoretical computer science, and in this paper I will use techniques developed by Alexander Sedoglavic in 2017 to produce new upper bounds for matrix multiplications of medium rank. I will begin by giving key definitions of algebraic complexity and tensor rank. Then I will revisit historical results obtained for upper bound and lower bound of the matrix multiplicative complexity from such authors as Winograd, Strassen, and Landsberg. Next, I'll explain the basic method developed by Sedglovic and how it can be useful in decomposing multiplication tensors and how it can be further improved on to have less loss of efficiency. Finally, I will give a comparison of all known lower and upper bounds with a summary of methods that have the potential to generate new asymptotically faster algorithms.

49. A Cost Benefit Analysis of Cyber Defense Improvements

Tung Thai Wentworth Institute of Technology
Advisor(s): John Haga, Wentworth Institute of Technology

In the past few years, several major cybersecurity attacks on supervisory control and data acquisition (SCADA) devices have been reported. Such attacks can result in damages to the economy and have an impact on society. In 2010, Ten et al presented an attack tree mode of impact analysis. We have implemented the attack tree structure developed by Ten, in concert with typical financial loss data to implement Monte Carlo techniques to generate a new cost benefit analysis of various security improvement scenarios. Time to attack is modeled as an exponentially distributed random variable obtained via maximum likelihood analysis; financial losses are modeled using regression to generalized logistic functions via gradient descent. Under these conditions, hypothetical future losses are simulated for a variety of intrusion scenarios and improvement schemes. Our model incorporates budgetary constraints in an effort to advise the prioritization of system improvements, and we compare the results of genetic and differential evolution algorithms in determining an optimal budget allocation.

50. Cellular-Scale Modeling of Oncogenic Proteins

Bernardo Hernandez Adame Massachusetts Institute of Technology
Amanda McAdams Washington University in St. Louis
Advisor(s): Liam Stanton, San Jose State University

Mutations in the RAS family of proteins have been implicated in roughly 25% of all human tumors and up to 90% in certain types of cancerous tumors, such as pancreatic cancer. RAS mutations can lead to overactive signaling in cells, which prevents cell death and leads to tumor growth. In order to better understand the dynamics of RAS protein interactions with the cell membrane and RAF proteins, a combination of atomistic data and a continuum scale model is constructed. The various interactions are incorporated into the model through a "*free*energy" functional, which describes the available work in this thermodynamic system. Furthermore, the evolution equations describing the changes in the membrane's lipid concentrations, the membrane's height, and the proteins' positions are derived using dynamic density functional theory. A sensitivity analysis is then conducted on the parameters of the protein-protein interactions due to the uncertainty in these values.

51. Donation Record Analysis for Baltimore Humane Society

Jennifer Weiler Towson University
Advisor(s): Alexei Kolesnikov, Towson University

This project was performed by a team of undergraduate students of Towson Universitys Applied Mathematics Laboratory. The primary goal of the project was to identify ways for the BHS to increase the donation revenue by analyzing the donation history data. My portion of the project was accomplished by cross-referencing the location of the donors with the U.S Census American Community Survey financial data and by providing estimates of the financial parameters for each of the donors. This task was performed in Python by computing the distances of each donors geocoded address to the two nearest county subdivisions centroids using the haversine formula. Weighted averages for each of the financial estimates affiliated with these centroids were added to each Maryland donors profile. With the additional data of the financial estimates of each Maryland donor, regression tree analysis was used to predict whether a Maryland donor is a possible high-contributing donor. A possible high-contributing donor is a Maryland individual who is predicted to have an average annual donation amount greater than or equal to 500, $butwhoseaveragedonationamountsthelastfiveyearswerebelow$500.

52. Music information retrieval using wavelets and machine learning.

Mitchell Will Western Connecticut State University
Leland Roberts Western Connecticut State University
Ralph Venezia Western Connecticut State University
Advisor(s): Xiaodi Wang, Western Connecticut State University

Entertainment firms utilize different methods for optimizing user's entertainment and to maximizing profit. The purpose of our research is to develop a new method for classifying music using wavelets and machine learning techniques, such as support vector machines, logistic regression, and neural network to identify a song's genre and recommend similar songs. To accomplish this we must gather a database of Songs. Songs take up lots of space to store, so we first

convert each song into wavelet domain. Next, we can reduce its dimension using principal component analysis, and extract the most important features using feature analysis. Lastle to classify songs, we will use either support vector machine or logistic regression or neural network, and combine with feature analysis and principal component analysis to find a match. From all this we can develop a match, we can then recommend other songs from our database with similar features. We are very confident that our state-of-the-art method is different that other research sources. This is because we are classifying and recommending songs based on the features of the song not just its genre making our method superior.

53. Computational Fact-Checking through Relational Similarity based Path Mining

Alexander Michels Westminster College
Advisor(s): Pamela Richardson, Westminster College

The volume of information today is outpacing the capacity of experts to fact-check it, and in the Information Age the real-world consequences of misinformation are becoming increasingly dire. Recently, computational methods for tackling this problem have been proposed with many of them revolving around knowledge graphs. We present a novel computational fact-checking algorithm, RelPredPath, inspired by and improving on the techniques used in state-of-the-art fact-checking algorithms, PredPath and Knowledge Stream. Our solution views the problem of fact-checking as a link-prediction problem which relies on discriminitive path model, but draws on relational similarity and node generality to redefine path length. This gives our solution the advantage of training on more specific paths consisting of edges whose predicates are more conceptually similar to the target predicate. RelPredPath shows performance at-par with other state-of-the-art fact-checking algorithms, but leads to a more robust and intuitive model for computational fact-checking. Work partially completed during the Research in Industrial Projects for Students program at UCLA's Institute for Pure and Applied Mathematics.

54. Theoretical Nanoparticle Light Scattering

Katlyn York Simpson College
Jacob Austin Simpson College
Kaylee grabarkewitz Simpson College
Advisor(s): Nicolas Rey-Le Lorier, Simpson College

We present an experimental method to dynamically determine the composition, in terms of shape and size, of a mixture of nanoparticles suspended in water. This method is based on the use of the Discrete Dipole Approximation (DDA) to predict the scattering pattern of light incident on the sample. In this report we present a general background to light scattering, including Maxwell's equations, Stokes vectors, and cross sections. The theory of the DDA is reviewed and applied to shapes constructed of silver nanocubes. We give examples of the application of this method to various sample types and discuss applications and limitations. Theoretical results are then compared to experimental data.

55. Obtaining the Thermophysical Properties of the Solids Using the Measured Distribution of Temperature Within the System of Contacting Solids at Uniform Heating

Scott Tilton Montana State University
Kyle Hammer California State University Chico
Advisor(s): Sergei Fomin, California State University Chico

This paper focuses on modeling and then determining thermal diffusivity and conductivity of an unknown material that is between a heat source and a material that is known. To begin, we obtain the heat equation with boundary conditions that this setup of materials forces. We nondimensionalize the system and then use Laplace transforms and geometric series representations to obtain an analytic solution. From this we then model sample data from this solution to our heat equation by minimizing a functional that depends on both parameters of conductivity and diffusivity of the unknown material. By minimizing this functional we are able to obtain the thermal conductivity and diffusivity of the unknown material. We also determine a method to find a minimum by minimizing one functional that depends on one parameter using an asymptotic approximation of our analytic solution as well as minimizing a second functional that depends only on one parameter. This second method is less computationally intensive and also produces accurate results on the unknown materials thermal properties. This research was done as an REU at California State University Chico under the direction of Sergei Fomin.

56. Defining the Relationship: Computer-Driven Characterization of the Binding of Host and Guest Molecules

Joyce Chew Calvin College
Advisor(s): Douglas Vander Griend, Calvin College

Host-guest chemistry has a variety of important applications in supramolecular chemistry, biochemistry, and inorganic chemistry. The binding of host and guest molecules is characterized by the stoichiometries in which they bind and the strength of that binding. Binding strength is quantified by binding constants and can be calculated via existing chemometric methods. However, the accuracy of these calculations depends on the selection of the correct stoichiometry before calculating a binding constant. This reaction stoichiometry determination is traditionally done by hand and is therefore inefficient and frequently overlooks nonstandard possibilities. Three optimization algorithms are applied to the simultaneous determination of the stoichiometries and binding constants of host-guest binding systems, and the suitability of each algorithm is evaluated based on speed and accuracy. Methods of searching discrete neighborhoods of optimized stoichiometric ratios to obtain chemically meaningful results are developed. Finally, a computational protocol for determination of stoichiometric ratios and binding constants of host-guest binding from spectrophotometric titration data is proposed.

57. Application of a Buffered Fourier Spectral Method

Monica Davanzo University of Central Arkansas
Advisor(s): Yinlin Dong, University of Central Arkansas

Standard Fourier spectral method is efficient for solving problems with periodic boundary conditions, but oscillations occur for problems with non-periodic boundary conditions. This can be corrected using a buffered Fourier spectral method. For non-periodic functions, a buffering polynomial will be added to the right end boundary, making it smooth and periodic on the boundaries, before applying FFT. Then the buffering zone can be removed to compute maximum error and order of accuracy. Using this method, the derivatives of non-periodic functions can be approximated and the solutions of select ordinary differential equations can be calculated and the error reduced from 10^{-5} to 10^{-11}.

58. Metachronal and Synchronous Propulsion at Low to Intermediate Reynolds Number

Shawtaroh Granzier-Nakajima University of Arizona
Advisor(s): Calvin Zhang, University of Arizona

The Scallop theorem states that, to achieve propulsion at low Reynolds number in Newtonian fluids, a swimmer must deform in a way that is not invariant under time-reversal. We exemplify the Scallop Theorem by modeling a swimming body with multiple swimming pedals at low to intermediate Reynolds numbers. The swimming body is modeled using the immersed boundary method, a widely used mixed Eulerian/Lagrangian framework for simulating the motion of elastic structures immersed in viscous fluids. Metachronal propulsion with an approximate quarter-period phase difference leading the cycle is found to be the most efficient in contrast with a time-invarient synchronous zero phase difference. This frequency-invariant stroke pattern is found to be the most effective and mechanically efficient paddling rhythm at low to intermediate Reynolds numbers.

59. Analyzing Rotavirus Using Game Theory

Robert Babac University of Guam
Jayson Morales University of Guam
Jacob Aquiningoc University of Guam
Advisor(s): Hyunju Oh, University of Guam

Rotavirus, a highly contagious virus transmitted via the fecal-oral route, is most commonly known to cause gastroenteritis, a disease inducing symptoms such as diarrhea, fever, abdominal pain, and vomiting in children under five. The virus, first discovered in 1973, currently causes 2 million hospitalizations and roughly 500,000 deaths per year. Two vaccines, Rotarix (RV1) and a vaccine given at birth (RV3-BB), have been shown to decrease the occurrence of severe gastroenteritis. By using a mathematical model, we analyze the transmission of rotavirus. With Game Theory, we construct a game theoretic model to determine the optimal vaccination strategies between both vaccines. In conclusion, we find the neonatal vaccine was not as effective in treating the virus compared to the Rotarix vaccine. NREUP at University of Guam

60. Tensor flattening approaches to estimate lower bound of small matrix multiplication tensor's border ranks

Yu Ma UC Berkeley
Advisor(s): Olga Holtz, UC Berkeley

Matrix multiplication efficiency lies at the heart of every computer algorithm. Strassen's algorithm initiated a new field of complexity theory looking for improvements on this problem. One popular approach in recent years is to evaluate the border ranks of tensor representations of matrix multiplications, written as $M_{\langle n,m,l\rangle}$. Unfortunately, even small sized problems such as determining the tensor rank of $M_{\langle 3,3,3\rangle}$ has not yet been solved. This project presents a detailed overview and applications of a newly emerged method, tensor flattening, that has shown to successfully assist evaluating lower bound of the border rank of $M_{\langle n,n,n\rangle}$. Furthermore, we specifically looked into the merits and limitations of Young flattening, proposed by Landsberg in 2013, with preliminary computational and theoretical investigation of its applications on small scaled rectangular matrix multiplication tensors. We end with a discussion of the implications of these results as well as the potential for generalization to other complexity problems.

61. Gone With the Flu

Sonali Vyas California State University, Fullerton
Kristina Rosete California State University, Fullerton
Advisor(s): Roberto Soto, California State University, Fullerton

Multiple strains of the influenza virus affect people every year, and every year a handful of people get infected with at least one of the many strains. A significant issue in the medical field is how to decrease the population of infected individuals. We created a model that could mathematically determine the rate at which vaccines for the influenza virus would have to be distributed to minimize the quantity of people infected. We accounted for the rate of contracting the influenza virus, the recovery speed, and the vaccination rates. We also considered the possibility of an individual getting infected with more than one strain. This model could help those in the healthcare industry create a more efficient process for vaccinating people and decreasing the number of those infected.

62. Decoding of Helberg Error-Correcting Codes for Multiple Insertion Errors

Emily Sandlin Rowan University
Advisor(s): Hieu Nguyen, Rowan University

Binary error-correcting codes have important applications in communications and data storage such as wireless networks and flash memory where error-correction is needed to recover corrupted information due to noise. Levenshtein codes are binary codes that can detect a single insertion/deletion bit error, and Helberg codes generalize Levenshtein codes to correct multiple insertion/deletion bit errors. We present new efficient decoding algorithms for Helberg codes to correct three insertion errors using binary search.

63. Modeling the Spatio-Temporal Dynamics of Interacting Fish Species in the Northeast Continental Shelf

Sara Amato Assumption College
Lauren Moore University of Kentucky
Kaitlin Ragosta Boston University
Advisor(s): Andrea Arnold, Worcester Polytechnic Institute

Knowledge of the population dynamics of marine species is vital to understanding ocean sustainability. This project aims to develop and analyze spatio-temporal single-species and multi-species models for studying fish population dynamics in the Northeast Continental Shelf, specifically Atlantic cod and Atlantic herring. We formulate partial differential equation models and integrodifference models that take into account species interactions between Atlantic cod and Atlantic herring. We determine a method to compare our single-species and multi-species models, to provide information to the Northeast Fisheries Science Center on whether either species would benefit from being assessed with a multi-species model. All models consider species' behavior, including seasonal migrations. We employ statistical approaches such as nonlinear filtering to estimate model parameters and quantify uncertainty in model predictions, comparing the results to synthetic data. This work was done as part of the REU program in Industrial Mathematics and Statistics at Worcester Polytechnic Institute.

64. Monte Carlo Simulation Using Wavelet filtering, Support Vector Regression, and Recurrent Neural Network for American Option Pricing

Srihita Mediboina Stony Brook University
Minyang Zhang UCLA
Yinqi Chen University of Connecticut
Tony Lee Choate Rosemary Hall
Advisor(s): Xiaodi Wang, Western Connecticut State University

Researchers have developed so many models for option pricing. Among such the Black Scholes model is the most popular model both in theory and in practice. As researchers concentrate more on the stochastic volatility as a factor of option pricing, Binomial Tree Model, Monte Carlo Simulation, Support Vector Regression, Neural Networks are used as methods to predict the option price, as well. However, there are some disadvantages in old models. For example, the exercise date is assumed to be the last day, while American options could be exercised before the exercise day. Furthermore, the old models based on limited factors. To gain more accurate results, we need to consider more and weight these factors differently. In this research, we provide a new model by integrating old models such as Monte Carlo simulation using wavelet filtering, support vector regression, and recurrent neural network that can predict the dynamic option price more accurately and compare the results from different models. We selected Apple, Facebook, Netflix, Tesla, and Google as database and focused on the first quarter of 2018.

65. Donation Record Analysis for Baltimore Humane Society

Kristian Brown Towson University
Advisor(s): Alexei Kolesnikov, Towson University

The Baltimore Humane Society is interested in increasing the total amount in donations they receive each year. Research involving their current donor database and donation totals is being conducted to help them achieve this goal. Part of this research includes conducting a statistical analysis to calculate and assign a numerical value to the organization's fundraising campaign. A t-test and a linear regression model is used to determine the relationship between the timing of fundraising events and the organizations monthly donation totals. This analysis allows the organization to see which fundraising initiatives bring in the most donations. It also shows how the increase in donations as a result of fundraising events impact the months following the event.

66. Gone with the Flu

Kristina Rosete CSU Fullerton
Sonali Vyas CSU Fullerton
Advisor(s): Roberto Soto, CSU Fullerton

Our project adapts a method from Optimizing the Allocation of Vaccines in the Presence of Multiple Strains of the Influenza Virus to calculate the best vaccination rate in order to minimize the number of people infected with the influenza virus. The three strains of type A during the flu season are H1N1, H3N2, H2N2. Our goal is to optimize the vaccine rates for each strain in order to prevent an epidemic. Our model illustrates how the typical flu season affects the population. Previous research has studied how to optimize the vaccination rate for only two strains from type A [2], thus in order to make the model more accurate, we included the final strain. With our model we will be able to better understand how to prevent flu epidemic resulting from these three strains.

67. Statistical Analysis and Geographical Clustering of Arrest Data for Los Angeles County

Cameron Hooper California State University, Fullerton
Advisor(s): David Uminsky, University of San Francisco

The work of the Los Angeles Police Department is something many sociologists and criminologists are intrigued by, but because of a lack of mathematical and statistical skills and clean, unbiased, and available data, their analysis is limited. In this study, we evaluated, interpreted, and analyzed a raw data set directly from the Los Angeles Police Department using advanced mathematical and statistical concepts. Our work entailed visualizing multiple variables using graphs and clustering, which allowed us to build an understanding of who is being arrested in Los Angeles County and how their residence plays a role. We used three clustering methods to aggregate similar neighborhoods based on the types of crime people were arrested for. By understanding the trends of arrests and the distribution of

variables in the data set, we conducted a unique analysis. We hope to see this work lead to ways to prevent predictive policing from contributing to disparities and bias towards certain racial/ethnic groups and communities. This research was conducted during the Mathematical Science Research Institute Undergraduate Program (MSRI-UP).

68. Modeling Climate Change in Togo, Africa

Jacob Englert Northern Kentucky University
Advisor(s): Andrew Long, Northern Kentucky University

The West African nation of Togo has experienced rapidly rising temperatures over the course of the past six decades. Continually rising temperatures have disastrous implications for agriculture, public health, and the general economy of Togo. We have constructed a regression model that captures the annual increase in temperature for all of Togo, based on monthly temperature data from ten different cities in the various geographic regions of Togo over the course of nearly 60 years. Singular Spectrum Analysis (SSA), an extension of the Singular Value Decomposition, is used to identify and thus incorporate seasonal fluctuations into the model, permitting us to isolate the underlying increasing trend in temperature - a trend that suggests well over 2 C of warming by the year 2100.

69. A Bilevel Optimization Approach to Variational Models

Brendan Gramp George Mason University
Advisor(s): Harbir Antil, George Mason University

We present a bilevel optimization approach to image denoising. The inner problem is the Total Variation (TV) based minimization algorithm and the outer problem identifies the regularization parameter. In the latter case two different objective functions are considered (i) machine learning based; (ii) statistics based. This research was done under the EXTREEMS-QED (Expeditions in Training, Research, and Education for Mathematics and Statistics through Quantitative Explorations of Data) program organized by the Department of Mathematical Sciences at George Mason University and funded by the National Science Foundation.

70. Recommender Framework for Lyme Disease Treatments

Catherine Wahlenmayer Gannon University
Eric Chen University of California, Los Angeles
Advisor(s): Deanna Needell, University of California, Los Angeles

Lyme disease is greatly understudied; however, patients have banded together to pool their data via the MyLymeData survey through LymeDisease.org to facilitate research. This presentation will discuss an adaptable framework created to recommend antibiotic and alternative treatments. The framework can also predict potential side effects from alternative treatments. It utilizes relevant portions of patient survey data to find medical regimens for unwell patients based on patients with similar experiences. The framework is adaptable to different questions within the context of MyLymeData or similar contexts as well as different classification methods. This research was conducted as part of the 2018 Summer REU at the University of California, Los Angeles.

71. A Statistical Model for Learning Movie Preferences

Jordan Turley Centre College
Zeyang Huang Centre College
Advisor(s): Michael Lamar, Centre College

We present a statistical learning model applied to the Netflix Prize problem, a 2006-2009 competition for developing an algorithm to predict customer ratings of Netflix movies. The Co-Occurrence Data Embedding algorithm (CODE) has previously been successfully applied to co-occurrence data, and with this work, we extend the modeling approach to Likert-scale data. We demonstrate this extended models utility using the Netflix data set to learn embeddings of users and movies based on the rating history. The embeddings are then used to compute a conditional probability distribution for each possible rating given a user-movie pair for which no rating is provided. This probability distribution is then used to predict an expected rating score for the pair. We evaluate the quality of the learned embeddings by comparing these predicted ratings with the actual ratings in test cases from the data set. The resulting algorithm is fast, simple, and intuitive, and its predictions match those of other more complex approaches.

72. The Impact of Network Structure on the Dynamics of Decision-Making Processes

Han Huang Swarthmore College
Advisor(s): Victor Barranca, Swarthmore College

In this study, we developed and analyzed a new network-based modeling framework for decision-making in tasks involving a large number of alternatives. We conducted thorough analysis on the most canonical network generalization—the all-to-all network. We analyzed the existence, uniqueness and stability of equilibria, corresponding to different decisions, as well as the reaction time necessary to make a decision. Next, we explored three additional types of topologies: regular, random, and small-world connectivity. Through extensive investigation, we concluded that regular and small-world networks excelled in achieving high accuracy. In addition, we studied the characteristics of two gain functions in integrating information: sigmoid and binary. We showed that steeper gain functions yield better accuracy at the price of longer response times, while the more gradual ones, besides being able to make decisions faster, are also more robust in the presence of noise or degradation of connections.

73. Hemoglobin Response to Higher Order Gene Interactions A Spectral Analysis Approach

Rosa Garza California State University, Monterey Bay
Advisor(s): David Uminsky, University of San Francisco

With the growing amount of genomics data, we need efficient and effective ways to analyze interactions of gene mutations. In this research, spectral analysis is used to orthogonally decompose a genomic data set in order to analyze higher order interactions between mutations and their effects in terms of frequencies. This approach offers insight into the effects that certain gene mutations groups have on hemoglobin while accounting for redundant information. After our analysis, we manually selected the mutation groupings that seemed to have significant effects on hemoglobin levels. Two-sample t-tests were then performed to see if our results had significance. The results from our research can be extended to analyses of the variations observed in other phenotypes. This research was done through the Mathematical Sciences Research Institute- Undergraduate Program (MSRI-UP) at UC Berkeley.

74. Dynamics of the Inextensible Inverted Flag with Piston-Theoretic Forcing Term

Varun Gudibanda Carnegie Mellon University
Advisor(s): Jason Howell, Carnegie Mellon University

In the inverted flag system a thin metal beam is fixed on one end and free to move on the other. Fluid flows from the free end to the fixed end, inducing motion of the beam. This model is of current interest in several applications, including in the design of piezoelectric energy harvesting devices. A full model of the system with an inextensible beam (i.e. of a constant length) considers the movement of the fluid and the beam separately, resulting in a system of PDEs that is computationally expensive and analytically difficult to investigate. Engineers looking at this system make simplifying assumptions on the interaction between the fluid and the beam using tools from Piston Theory, resulting in a single PDE. The main objectives of the project were to determine the influence of the Piston-Theoretic assumption on the dynamics of the inverted flag and determine parameters that influence critical behavior of the system. By conducting a modal analysis and truncating higher order terms, we found that for wind speeds below a critical value the inverted flag will eventually damp to an equilibrium state, and for higher wind speeds, the beam converges to a deflected state.

75. A Comparison Between Greedy Coordinate descent and The Parametric Simplex Method in Machine Learning Optimization

Adam Kelly Princeton University
Advisor(s): Robert Vanderbei, Princeton University

This research tests the implementations of two different optimization algorithms applicable to the LAD-LASSO regression problem. The benefits of LASSO regression have been already studied on extremely large datasets with p¿¿n. The specific benefits from LAD-LASSO stems from both its efficiency to compute, as well as its improved ability to ignore outliers. Both due to it having an objective function that can be formulated as linear, and its simple directional derivatives linear programming methods, and greedy coordinate descent methods can be utilized effectively. As a result of the two main optimization techniques having widely varying methodologies, guarantees, and implementation considerations they are treated separately before the final comparison. Where the parametric simplex method offers

a complete understanding of the error at each possible value of lambda, for known values or a few values greedy coordinate descent promises greater scalability. There are techniques for using greedy coordinate descent in parameter selection in conjunction with cross validation which are discussed, and compared to the parametric simplex method.

76. Numerical Method for Solving the Far Field Refractor Problem

Sekou Rowe Howard University
Harena Yemane Howard University
Advisor(s): Henok Mawi, Howard University

The far field refractor problem in geometric optics is an inverse problem which deals with finding a refracting surface (lens) that is capable of reshaping a light beam from a given point source shining through an aperture cone Ω with prescribed illumination intensity, so that it covers a prescribed region Ω^* in the far field with a predetermined intensity distribution. The existence and uniqueness up to dilations of such surfaces has been studied by using optimal mass transportation techniques and also by using approximation by semi ellipsoids assuming some geometric conditions on Ω and Ω^*. A numerical algorithm to approximate such a surface is demonstrated in the work of R. De Leo, C. E. Gutiérrez and H. Mawi (2017). In our work we exhibit an alternative method to numerically approximate the solution to the far field refractor problem.

77. Detection of Atrial Fibrillation in Electrocardiograms via Persistent Homology-based Features

Esteban Escobar California State Polytechnic University, Pomona
Advisor(s): David Uminsky, University of San Francisco

Mathematical Sciences Research Institute Undergraduate Program Atrial Fibrillation is a common heart condition affecting nearly thirty-three mil- lion people worldwide. Atrial Fibrillation can lead to erratic heart behavior that can cause blood clots, heart failure, and even death. It is difficult to diagnose due to distortion in the cardiac rhythm caused by spasms. In response, the Phy- sioNet/Computing in Cardiology launched a challenge in 2017 to design models that can classify electrocardiogram (ECG) readings and accurately detect Atrial Fibrillation. We use persistent-homology based features to classify ECGs and accurately detect Atrial Fibrillation.

78. Reconstructing Elastic Grid Structure from Noisy Landmark Points

Yuepeng Yang Carnegie Mellon University
Advisor(s): Dejan Slepev, Carnegie Mellon University

Exactly recovering grid structure is important to a number of applications in image processing. The ability to reliably and precisely recognize a deformed grid is essential for deformation tracking. Because of the noise and damage in the images, it is not always possible to obtain landmark points that accurately represent the position of the grid vertices. Consequently, an important part of this problem is to robustly reconstruct an elastic grid structure with noisy and potentially missing or erroneous vertices. We design an approach for elastic mesh registration which rewards agreement with the landmarks and the smoothness of the deformed mesh. To eliminate the effect of misleading landmark points and reliably recover the mesh structure, we develop a randomized algorithm that starts with small patches and progressively propagate to larger meshes. We also employ spline interpolations to complete holes where landmark points fail to offer any reliable information. In addition, we are able to build time series alignment procedure that tracks the change of the mesh structure over time.

79. Pricing TYVIX Options Using a Risk-Neutralized Historical Distribution

Montgomery Fischer University of Georgia
Rajita Chandak Brown University
Jonathan Ladd Oberlin College
Anthony Sisti University of Connecticut
Advisor(s): Marcel Blais, Worcester Polytechnic Institute

Volatility indexes provide a measure of unpredictability in the future behavior of financial markets. Two such indexes, the VIX and TYVIX, were created by the Chicago Board Options Exchange (CBOE) to quantify expected volatility in the US stock and bond markets, respectively. Over the last 15 years, volatility derivatives have gained popularity among investors for hedging risk and diversifying investments. While options are not yet publicly traded on the TYVIX, there

is interest in finding fair prices for TYVIX options that are traded over-the-counter (OTC). We first develop a pricing model for exchange-traded VIX options that may be validated using current market prices, and then extend the model to price TYVIX options. Building on Zou and Derman's methodology in "Strike-Adjusted Spread," the historical distribution of returns is transformed to satisfy risk neutrality at a minimum loss of information. Using this risk-neutralized return distribution, the fair market price for an option may be predicted by calculating its expected future payoff. Our model was developed in Worcester Polytechnic Institute's REU in Industrial Mathematics and Statistics.

80. Continuous Opinion Dynamics on an Adaptive Network

Xinyu Zhang The College of William and Mary
Advisor(s): Leah Shaw, The College of William and Mary

We present a model for opinion dynamics on a network. Opinions are assumed to be a continuous variable, reflecting a possible spectrum of real world opinions. A nodes opinion is updated to become more similar to a randomly selected neighbors opinion, provided that the neighbors opinion differs by less than a threshold. Initially considering a static network, we establish criteria to determine whether consensus or clustering will be the outcome of the dynamics and on what time scales these states will be reached. Next, in contrast to the static networks with fixed structures, we incorporate the changing nature of the interpersonal relations in real-life social networks. In addition to the opinion dynamics, links that do not communicate due to divergent opinions may be broken with some probability and new links established. In this way, the network changes in an adaptive manner, which combines the topological evolution of the network with dynamics in the network agents. We consider how adaptation affects the final state of the system.

81. Assessing the Role of Prescribed Painkillers and its Impact on Opioid Epidemic

Abel Reed Dixie State University
Advisor(s): Vinodh Chellamuthu, Dixie State University

The United States is currently facing an opioid crisis that causes more than 40% of the overdose deaths. Over the past decade, the population of opioid addicts in the U.S. has increased causing a negative impact on adolescence life style. To understand why opioid addiction is rising in our society, we need to better understand the relationship between prescribed painkillers and the rate at which opioid addicts enter rehabilitation programs. We have developed a mathematical model that utilizes a system of nonlinear differential equations to investigate addictive phenomena in the opioid epidemic. We solved the model using a non-standard finite difference numerical scheme. Our numerical simulation results show a broad view of what factors are directly contributing to the growth of the addicted population within our society. We also plan to incorporate seasonality into the model to reflect the variability in prescription rate within opioid dynamics. Our model could be used by hospitals and rehabilitation centers to discern when their patients are most liable to become addicted, as well as what factors are directly contributing to opioid addiction.

82. Wildfire: A Mathematical Model Analyzing the Effects of Fire Damage

Jake Skinner Dixie State University
Advisor(s): Vinodh Chellamuthu, Dixie State University

For Californians, wildfire season is no stranger. As the intensity and devastation has increased over the last several years, even those accustomed to the fires are growing weary. The financial burden on insurance companies and home-owners is outstanding and continues to grow with each season. One way to help with this burden is to understand how fire spreads in certain areas based on various factors and which areas are more at risk for fire damage. Knowing this helps set accurate premiums in order to cover home values for their clients. Based on information gathered from the U.S. Census Bureau and other reliable sources, we have developed a mathematical model based on the traditional SIR model. The SIR model has typically been used for the spread of infectious diseases; what is fire but a disease or virus for structures. The Thomas fire in Ventura County during 2017s fire season was used to gather data and create this model. Our model can be used from an actuarial standpoint for insurance companies to better predict and analyze damages that can occur.

83. Fluid Dynamic Modeling of Vapor-to-Particle Reaction Systems

William van Noordt Colorado State University
Advisor(s): Patrick Shipman, Colorado State University

Natural and man-made processes contribute to the formation of atmospheric aerosols. By scattering sunlight, aerosols have a significant influence on climate and climate change. They also have adverse health effects on large populations.

Aerosols nucleate and grow rapidly. In this study, we combine experimental results with mathematical fluid-dynamical models to study the relationship between fluid dynamics with the nucleation and growth processes. This research was carried out at Colorado State University.

84. Tanh Activations in Image Classification

Jonathan Scott Macalester College
Advisor(s): David Uminsky, University of San Francisco

Our research focuses on applying deep learning architectures with respect to classifying images that have been corrupted by Poisson noise. We implement our method using Pytorch on the MNIST dataset by using a variety of neural network architectures. We incorporate convolutional neural networks (CNNs). Many recent deep learning architectures implement the Rectified Linear Unit, ReLU as a default choice. Classic activation functions such as Tanh and sigmoid as well as new variations of ReLU and other functions have provided a growing list of options for users. We study the importance of choosing an activation function, however , instead of proposing a new activation function, we study the relationship between the most commonly used ones. We propose a two-parameter, trainable Tanh activation function which we call TAct. TAct is a family of activation functions, which exactly contains classic functions such as Tanh, Sigmoid, and more recently Swish. This work was done during MSRI-UP 2018 at the Mathematical Sciences Research Institute.

85. Comparative Study for the Lane-Emden Equation in Stellar Configuration to its Higher Order dynamics

Breanna Shi Stetson University
Kaitlin Harding Rochester Institute of Technology, Rochester, NY USA
Abbas Rehmani University of Wisconsin-Madison, WI USA
Advisor(s): Fazal Abbas, Stetson University, DeLand Fl USA

In this study, we consider the Lane-Emden equation of the first kind which arises in the study of stellar structures. The higher order structure of this equation in the non-iterative setting has been proposed and verified through mathematical induction. We develop the Homotopy Analysis Method (HAM) algorithm to obtain the convergent series solutions to the model equations. Choice of the initial value, nonlinearity, and the choice of a linear operator in the construction of HAM algorithm affect the solution radius of convergence. Analytical approximations are compared with the Pade approximant to a power series solution and Runge-Kutta-Fehlberg method (RKF45). The numerical comparison shows that the traditional series solution is convergent in that of a domain where the solution is positive.

86. The Effect of pH and Aggregation on Anthocyanin Color: A Mathematical and Experimental Analysis

Rachael Tindal Colorado State University
Advisor(s): Patrick Shipman, Colorado State University

Red, blue, and purple colors in a variety of flowers and fruits are due to a class of plant pigments called anthocyanins. Anthocyanins undergo a series of complex color changes, influenced by factors such as pH and concentration. Through a series of experimental studies and mathematical models, we study how anthocyanin color changes are affected by their aggregation. There is abounding interest in anthocyanin color due to its applications of giving color to flowers, fruit and wine, and attracting pollinators to flowers. Possible uses in industry include dyeing various foods, beverages and clothing items.

87. privacy-preserving deep learning and support vector regression methods

Mingyang Zhang UCLA
Kevin Li Somers High School, NY
Advisor(s): Xiaodi Wang, Western Connecticut State University

A statistic is a single measure of some attribute of a sample which deals with the collection, organization, analysis, and interpretation of data. Suppose a trusted party gathers sensitive information from a large number of respondents, with the goal of learning statistical facts about the underlying population. The goal of privacy-preserving data analysis concerns to release statistical information without private data from leakage. In recent years, many researchers have

shown that anonymizing data will not preserve privacy of data. To achieve the goal, some robust concepts of privacy-preserving such as k-anonymity and l-diversity have been proposed. However, even these methods cannot prevent private information leaking from attacks. In an extreme case, the attacker might know the contents of all but one of the rows in the set. To combat such background attacks, we propose a practical system that enables multiple parties to jointly learn some accurate neural-network and support vector regression models for a given objective without sharing their input data-sets.

88. Modeling Vapor-to-Particle Ammonium Chloride Band Formation

Austin Fearn Colorado State University
Advisor(s): Patrick Shipman, Colorado State University

Vapor-phase hydrochloric acid ($HCl_{(g)}$) and ammonia ($NH_{3(g)}$) undergo a complex series of nucleation, diffusion, convection, and reaction events to form aerosol ammonium chloride ($NH_4Cl_{(s)}$). When the vapors counter-diffuse in a glass tube, these processes form an intricate pattern of bands that appear over time on the tube wall. Color variation occurs in the bands and stems from variation in particle size distributions. We extend a partial differential equation model for band formation to build the first model which accounts for particle size distributions over space and time. Using the experimental initial conditions, numerical simulations of the model produce $NH_4Cl_{(s)}$ concentration spikes and particle size variation along the tube over time. Vapor-to-particle reactions like this one occur naturally and the resulting aerosols and their formation have far-reaching effects on climate, weather patterns, and even biochemical processes. Mathematical systems like this are vital to a deeper understanding of these types of phenomena. This research was conducted during the 2018 Colorado State University Mathematics REU.

89. Perceptual Image Hashing of Video

Linda Beverly California State University, East Bay
Advisor(s): Shirley Yap, California State University, East Bay

We investigated perceptual image hashing techniques in order to improve upon existing methods for video data analysis. A perceptual image hash is a digital fingerprint obtained from an image or video data based on its visual appearance. This fingerprint may then be used to help identify other similar images or videos. We developed several different algorithms incorporating various techniques including Discrete Cosine Transform, Locally Linear Embedding, and kernel methods to perform perceptual image hashing. This work was supported by the Cal State East Bay Center for Student Research, LSAMP, and CREU. LSAMP is a National Science Foundation supported program. CREU is a project of CRA-W and is supported by the National Science Foundation.

90. Wavelet and Machine learning based Malware Detection

Shuo Mu United Experimental School of AHSJU and Livon
Advisor(s): Xiaodi Wang, Western Connecticut University

Malware is combination of two words from Malicious and Software. Malware has a huge impact on the information network spaces safety. Since 1983 the first person discovered a computer virus, malware technology presents the trend of rapid development and the characteristics of complex types, which are applied at all levels, posed great challenges to the confidentiality, integrity and availability of information and information systems. In this search project, we develop a new Malware detection algorithm base on wavelet and machine learning. We first setup databases for training and tasting. We then apply wavelet transform and Principle Component Analysis (PCA) for dimension reduction and feature extraction, we then create classifiers using logistic regression, Support Vector Machine (SVM), and Neural Network (NN) to detect malware.

91. How the application of Particle Swarm Optimization may help in the fight against cancer by using data from time sequences of medical images to determine the efficacy of a cancer treatment

Kao-Pu Chang Virginia Military Institute
Advisor(s): Jessica Libertini, Virginia Military Institute

The application of Particle Swarm Optimization may help in the fight against cancer. By using data from time sequences of contrast-enhanced medical images, we aim to assess the efficacy of a cancer treatment. This process requires the identification of four blood-flow parameters which measure the level of angiogenesis, which relates to

tumor growth. Nevertheless, in reality, we won't have the value of these four blood-flow parameters, we only have the concentration of contrast-agent. Therefore, we will have to do an inverse problem to access the four blood-flow parameters. This project explores the viability of Particle Swarm Optimization as an approach for recovering the four parameters.

92. Web search engine based on wavelet transform, Principle Component Analysis, and Support Vector Machine framework

Yushen An United Experimental School of AHSJU and Livon
Advisor(s): Xiaodi Wang, Western Connecticut University

Search engine, which has been widely used for various layers of internet applications, has encountered the invasion of malicious web pages, affect our daily work and learning efficiency. This research project focuses on creating a new search engine algorithm. We first collect the tens of millions to billions of web pages on the world wide web, and every word on a web page (i.e., keywords) indexes, then set up index database of full-text search engine, this "data collection framework" new optimization scheme is put forward, aimed at setting up a huge database as training and test sets. We then apply Wavelet Transform and Principle Component Analysis (PCA) for dimension reduction and feature extraction step by step with the Support Vector Machine (SVM) classifier for multiple alternative data base, so that the user can get what he/she wants when using a search engine to the target range.

93. Wavelet based spam filtering using Machine Learning

Ruoxi Wang Affiliated High School of Northeaster Normal University
Tangjian Wang Affiliated High School of Northeaster Normal University
Advisor(s): Xiaodi Wang, Western Connecticut State University

Nowadays, emails and smart phones have been widely used, and become the fastest and most economical ways of communication at present. However, the increase of users leads to a sharp increase in spam. Most of us may be confused and hurt by useless and even cheating information, which will affect our work efficiency and dally life and bring huge troubles to us. In this research project, a spam detection method combining wavelet transform and principal component analysis (PCA) is proposed to reduce dimensions, compress and extract typical features of the data set. Then, support vector machine (SVM) and Neural Network (NN), based on different kernels and transfer functions are adopted as the classifiers to improve accuracy and efficiency, including wavelet kernel and transfer function.

94. Wavelet denoise based data transmission method

Alexander Isaacson Western Connecticut State University
Advisor(s): Xiaodi Wang, WCSU

Wireless data transmission protocols are prone to surrounding radio noise and must send extra data used for error detection and correction to ensure that the data was received correctly. This research project explores the viability of using wavelet transforms to encode the noisy data into the waveform itself then apply the denoise procedure to remove the noise. Such a protocol must be easy to implement in hardware and resistant to common sources of noise.

95. Floquet Hofstadter Butterfly on the Kagome Lattice

Ariel Barr University of Texas at Austin
Advisor(s): Gregory Fiete, University of Texas at Austin

Hofstadters butterfly refers to the fractal energy spectrum of the Harpers equation, which occurs when a magnetic field distributes the energy levels of a lattice into a Cantor set. Lattices with Hofstadter spectrum exhibit quantized Hall conductance, experiencing topological phase changes under the influence of a periodic driving potential, and producing pairs of counter-propagating chiral edge modes. Materials exhibiting edge modes hold promise for future applications in high-temperature superconducting devices, as well as for quantum computing. We use Floquet formalism to theoretically study the influence of periodic driving potentials on the Hofstadter energy spectrum of Kagome lattices. Further, we study the influence of periodic driving potentials on the topological invariant (Chern number) of the lowest band in the off-resonance regions. Our theoretical model is coded in Fortran and run on the University of Texas TACC supercomputers. This work is currently under review at Physical Review B, and can be found at arXiv:1808.02057. We gratefully acknowledge funding from Army Research Office Grant W911NF-14-1-0579, NSF Grant No. DMR-1507621, and NSF Grant No. DMR-1720595.

96. Independent Research Experience on Floating Bodies

Jessica Vasquez Stanislaus State
Kao Chee Vang Stanislaus State
Marisol Miranda Stanislaus State
Advisor(s): Kenneth Hoover, Stanislaus State

We begin with a solid sphere of uniform density that is less than water, with the top portion of the sphere cut off. When we tilt the solid at an angle between 0¡¡90 and place it in water, the truncated sphere will rock back and forth until its axis is perpendicular to the surface of the water. The goal of our research is to understand why this happens. To find the solution, we must find the center of mass of three separate portions: the entire solid (C), the section under the water (Cu), and the section above the water (Ca). Using multivariate calculus, the rotation matrix formula, and Archimedes Principle, we are able to obtain this information. By this process, we also discover that all three centers of mass are contained in a line. We note that the force of gravity pushes down on Ca while the force of the water acts on Cu . This relation results in the solid rocking until it reaches equilibrium and its axis is perpendicular to the water surface.

97. A Rigorous Analysis on Miller and Volz's Edge-Based Compartmental Model

Sabrina Mai University of Central Florida
Advisor(s): Zhisheng Shuai, University of Central Florida

Edge-based network disease models, in comparison to the classic mass action model, better capture social factors affecting disease spread such as contact duration and social heterogeneity. We reason that, in Joel C. Miller and Erik Volz's edge-based model, there should exist infinitely many equilibria rather than only an endemic and disease-free equilibrium as there does not exist any changes in demographic in their model. We then modify their network model by relaxing some assumed conditions and factor in a dependency on initial conditions. We find that this modification still accounts for realistic dynamics of disease-spread (such as the probability of contracting a disease based off your neighbors susceptibility to the disease) based on $\mathcal{R}_0$. Specifically, if $\mathcal{R}_0 < 1$, then the infection dies out; while if $\mathcal{R}_0 > 1$, then there is an epidemic. We also show that an approximate reduction of our model still concurs with the behavioral results of Miller and Volzs model.

98. Convex Neural Code in Low Dimensions

Zejun Gao Colby College
Shuofeng Xu Colby College
Advisor(s): Nora Youngs, Colby College

Our brain can process huge amount of information coming from its ambient environment. However, it does not actually "see" or "hear" the world, and it can only extract information about the external world through neural codes, combinatorial objects which represent neural activity. Our investigation aims to shed light onto this process by focusing on neural codes of place cells. More specifically, we want to know to what extent neural codes can encode dimensional information about the organism's ambient environment. Using graph theoretical approaches, we provide a novel method to characterize neural codes that can be convexly realized in 1 dimension, and algorithmically identify certain obstructions to convexity in 2 (and higher) dimensions.

99. An Overview of the World Food Security and Policy Making

Shang-Cheng Su Virginia Military Institute
Advisor(s): Jessica Libertini, Virginia Military Institute

The goal of this research is to build metrics to examine what countries have food security and what countries do not have, and to develop food policies for countries that may have potential problems on their food security. With our metrics, we find that there are two categories (well developed or underdeveloped) of countries that are actually having potential problems on their food security. For the first category, countries like Singapore that have great economic development but highly rely on importation of food from their allies. As a result, those countries may encounter problems if they dont keep good relations with their allies. On the other hand, countries like Iran that are neither producing enough food, nor having good relations with other food producing countries may need to institute better food policies to avoid food security problems.

100. Characterizing Uncertainty in Carbon Sources/Sinks using Bayesian Methods

Siona Prasad Thomas Jefferson High School for Science and Technology
Advisor(s): Chinmay Kulkarni, MIT

Carbon dioxide (CO2) emissions, especially from large cities have resulted in the build-up of greenhouse gas concentrations responsible for climate change. Existing sensor technology is too expensive for large-scale use in urban areas, and cannot distinguish between anthropogenic and biogenic emissions. In this paper, we develop a low-cost in-situ sensor for direct CO2 measurements and a spectral imaging system to remotely monitor vegetation phenology. The sensor and camera were installed on a low-powered drone to serve as a mobile platform for accurate measurements and Bayesian models used this data to estimate emission inventories and model uncertainty. The predicted CO2 emission inventory for Washington DC showed a large contribution from the transportation sector. In summary, we demonstrate a methodology to measure and monitor city-wide CO2 emissions by combining low-cost in-situ measurements, multi-spectral imaging, small drone technology and puff modeling, and taking the first step to targeting specific greenhouse gas sources and reducing overall emissions.

101. Automatic Classification of Anthropological Bone Samples

Pedro Angulo-Umana University of Minnesota
Advisor(s): Peter Olver, University of Minnesota

The problem of posthumously identifying cause of death is of major interest in anthropology. If a reliable method of classifying bone fragments can be found, it can be used to characterize anthropological sites. A primary method used by modern anthropologists is break face analysis, whereby the way that a given bone sample is broken is used to draw conclusions about the agent of breakage. In particular, the angle at which the bone is brokenthe angle formed by the bones exterior surface and the plane of bone exposed by the breakageis thought to depend on whether the bone was modified via a localized impact, such as a primitive weapon, or by steady pressure, such as by a carnivore extracting meat. However, these measurements have usually been made via physical instruments, which can introduce a plethora of errors into the analysis. In this project, we automate the process of characterizing break angles by computing differential geometric quantities via robust curvature estimators on three-dimensional models of bone samples. These quantities are used to develop machine-learning classifiers that can allow anthropologists in the field to reliably classify bone samples.

102. Modeling temperature of solar panels

Abel Asfaw LaGuardia Community College
Andrea Martinez LaGuardia Community College
Advisor(s): Malgorzata Marciniak, LaGuardia Community College

Our objective for this is project is to analyze how the change in temperature affects the voltage of flat and curved monocrystalline solar panels. The project will be done at the rooftop of LaGuardia Community College and it consists of two solar panels: one unplugged and the other panel is plugged to a battery. We measure the temperature of the surrounding ambience and of the solar panels subject to weather conditions and use tools like Labquest and thermostat to collect data. We plan to apply newtons law of heating and cooling to determine the rate of change of the temperature and how it is affecting the solar panels. All the data collected will be analyzed through MATLAB and hope to see how our results could be applied to geometry of solar panels project which investigates the shape of panel and its exposure to the sun under different time of the year.

103. Quantifying and Managing the Uncertainty in Piecewise-Deterministic Processes

April Nellis University of Maryland, College Park
Tristan Reynoso University of Central Florida
Advisor(s): Alexander Vladimirsky, Cornell University

In piecewise-deterministic processes (PDP), the state of a finite-dimensional system evolves continuously, but the evolutive equation may change randomly as a result of discrete switches. A running cost is integrated along the corresponding piecewise-deterministic trajectory up to the termination to produce the *cumulative cost* of the process. We address three natural questions related to uncertainty in cumulative cost of PDP models: (1) how to compute the Cumulative Density Function (CDF) when the switching rates are fully known; (2) how to accurately bound the CDF when the switching rates are uncertain; and (3) assuming that the PDP is controlled, how to select a control to optimize

that CDF. In all three cases, our approach requires posing a (weakly-coupled) system of suitable hyperbolic partial differential equations, which are then solved numerically on an augmented state space. We illustrate our method using a simple example of trajectory planning under uncertainty. Done as an REU project at Cornell University.

104. Mathematics of Nested Districts: The Case of Alaska

Cara Nix University of Minnesota
Advisor(s): Moon Duchin, Tufts University

A great deal of recent attention has been given to the problem of detecting gerrymandering using mathematical and statistical tools. Many states favor plans that keep counties and cities intact rather than splitting them between districts, raising possible worries about whether the counties and cities themselves could be redrawn to favor an agenda. We focused on a class of redistricting principles called nesting rules, which require or encourage that members of the state-level House or Assembly be elected from districts that nest inside of the state Senate districts. In most cases, these rules dictate that each Senate district should be composed of exactly two adjacent House districts, as in the state of Alaska, in which 40 House districts are paired into 20 Senate districts. We will consider the scenario in which House districts are fixed first, and subsequently paired into Senate districts. Since we know of no rules requiring House districts to be set before they are paired, we will also consider varying the underlying House map. Across all these scenarios, we will discuss when and why the choice of pairing, or perfect matching, can have a sizable impact on electoral outcomes.

105. Using Survey Data and Mathematical Modeling to Prioritize Water Interventions in Developing Countries

Jordan Spencer Brigham Young University
Jane Cox Brigham Young University
Konnor Petersen Brigham Young University
Advisor(s): Tyler Jarvis, Brigham Young University

We describe a method for combining the World Health Organization's cost-effectiveness analysis with country wide survey data in order to construct an ordered ranking of the areas within a given country which have the highest need for a more reliable water source, it also includes the optimal method of water intervention. We also address a key problem in the charitable water sector: while survey data is available, due to privacy issues, much of the geographical and spatial data is lost or confounded. This disconnects the information from the locations in which they were found, making the data largely unusable. To overcome this, we propose using a combination of Voronoi modelling and gamma distributions to estimate an accurate representation of the data, allowing charities to overcome the lost information and increase their ability to use the available data. This method has been tested on the countries of Namibia and Madagascar and should be applicable to many more.

106. Investigating the Impact of Marijuana Dispensaries on Crime

Roberto Hernandez California State University, Fullerton
Advisor(s): Laura Smith Chowdhury, California State University, Fullerton

Federal and state laws vary on the legality of medicinal and recreational marijuana use. We investigate the claims that marijuana dispensaries are areas of high-risk; meaning that because of the nature of the establishment, there is a higher risk of encountering illegal activity in its near vicinity. We focus on the region of Portland, OR in Multnomah County during the time frame where new legislation has been implemented, reducing the penalties for marijuana related infractions. We wish to investigate whether these legislative changes influence the total amount of crime and the crime surrounding those establishments with Kernel Density Estimates, histograms, and cluster analysis. We observe there is a general increase in crimes against person and property. However, this trend is also seen in Louisville, KY where all marijuana use is illegal. Therefore, we cannot conclude the new legislation has impacted crime.

107. On the Shape of Large Soap Bubbles

Breanna McBean California State University, Fullerton
Advisor(s): Nicholas Brubaker, California State University, Fullerton

The behavior of bubbles has intrigued both physicists and mathematicians for over a hundred years. Much of the intrigue comes from the inherent ability of a soap-bubble to solve the well-posed mathematical problem: Given a bounded volume, what is the shape of the boundary surface with minimal area? In this classical problem, which was

first formulated by nineteenth-century physicist Joseph Plateau, the weight of the soap film is neglected. However, as the size of the bubble grows, macro-scale effects such as gravity, become more important and the shape of the surface can change significantly. We seek to characterize these qualitative changes via the following modified problem that captures the behavior of both small and large soap bubbles: Find the surface that minimizes the sum of surface and gravitational energy and encloses a specified volume. Additionally, this question is intimately related to the construction of heavy surfaces and provides practical insights into the shapes of energetically-optimal domes.

108. Threshold optimization in multiple binary classifiers for extreme rare events using predicted positive data

Edgar Robles University of Costa Rica
Fatima Zaidouni University of Rochester
Advisor(s): Aliki Mavromoustaki, University of California, Los Angeles

Classification on imbalanced datasets is a challenging problem where a high rate of correct detection is required in the minority class. We analyze the output of binary classification models used by Google, where the inputs are categorized as either positive or negative against a threshold. In rare-event problems, positives have a frequency under 0.1% and it is expensive to estimate all documents. Therefore, the problem is reformulated using the correct labels [true positive (TP)/false positive (FP)] on a sample of the predicted positives, as determined by human raters. It is important to pick an operating point (OP) on the TP/FP fitted curve whose position is adjusted to return the cost for one additional TP document in terms of the number of FP. We propose two solutions to select an OP by maximizing the area under the curve (AUC): a graph-based and an analytic approach. The graph-based approach selects an optimal path in the threshold space that is then converted to a curve in the TP/FP space. The analytic approach minimizes a cost function. Our approaches improve over existing solutions by offering a business interpretation to the OP. This was researched at the RIPS-LA REU.

109. Optimization of the power output of a micro-wind turbine

Jonathan Granada LaGuardia Community College
Advisor(s): Malgorzata Marciniak, LaGuardia Community College

The optimization on the geometric design of a Wind turbine is being discussed on this research. Using the blade element momentum (BEM) theory, which dived the wing in certain number of segments and calculate the power on each segment, then add them up, and Genetic Algorithm (GA) which is a method to solve optimization problems based on natural selection, just like evolution. A MATLAB program was design to pick a pitch angle and chord length at random within a given parameter, the resultant coefficient of power is what is used to determine in every generation if the resultant is improved, if it is, then it is a new starting point with the pitch angle and chord length. after running the program for 7000 generations using a NACA airfoil 4412, the resulting coefficient of power is at an average of 0.498257.

110. Cartographic Coordinate Conversion for Stellar Navigation

Austin Kreulach University of Arkansas
Advisor(s): Saad Biaz, Auburn University

Charting the earth is a difficult task. Early cartographers lost precision or valuable properties when they stretched and squashed the earth onto a flat plane, but now the question is how to map a slightly flattened spheroidal earth onto a perfectly spherical globe. Modern systems use geodetic latitude, a system that accounts for the fattening at the equator. This project originally created a system for converting to planar coordinates as part of autonomously navigating vehicles by the stars, the poster will focus on the details of the coordinate conversion and comparing the effects of the various standards of geodetic latitude on the final outcomes. This work was done at the Auburn University REU site for smart UAVs.

111. Nonholonomic Motion Planning for Self-Driving Cars

Samuel Schmidgall George Mason University
Advisor(s): Anton Lukyanenko, George Mason University

RRT* is a motion planning algorithm that uses incremental sampling to find an optimal motion plan. Unlike the algorithm RRT, from which RRT* originated, RRT* converges to an optimal solution. We present a novel application

of RRT* where the paths of multiple self-driving cars with nonholonomic constraints are computed in order to most efficiently maneuver the cars collaboratively so that they can each reach their end destination.

112. Random Walks and the Heat Equation

Tait Weicht Seattle Pacific University
Advisor(s): Jeffrey Schenker, Michigan State University

Random walks are processes useful for understanding a variety of stochastic behaviors. We explored a directed random walk model on the 2D integer lattice with particular interest in the stopped process that models capturing a walker at the origin. We derived a closed form for the probability of capture starting within n steps for a walker starting at (x, y) and related functions. In addition, we found analogous expressions for a Brownian mover and proved that these were asymptotic to those for our directed walker model. Numerically, we found evidence that the probability of trapping our directed walker is very well approximated by a re-scaled trapping probability for a Brownian mover even on relatively small timescales. We derived the scaling parameters from a single parameter in our model the turning probability of the directed walker. We also explored the potential applications of this relation and suggest a method for estimating population density based on fitting a class of curve we have derived from our analysis. We discuss the practicality of using these techniques to estimate population density in situations where capture-recapture techniques are not viable.

113. Constrained Optimization Problem with an Application to Folding

Jasmine Camero California State University, Fullerton
Erica Ward California State University, Fullerton
Advisor(s): Nicholas Brubaker, California State University, Fullerton

Optimization problems are ubiquitous in everyday life. In most physical situations, there is likely an involvement of conditions that enforce constraints on the optimal solution. In the current work, we study a mathematical model that will accurately predict when a paper structure will fold onto itself after a drop of water has started to evaporate from within it. By scaling down to a two dimensional cross section, weve analyzed the relationship between the angles with respect their vertical axes and constructed a model to measure the energy that is emitted from the changing positions. Through analysis, we have been able to determine the minimum amount of energy required to move the structure into its desired state.

114. Differentiable Geometric Invariants for Bone Fragment Refitting

Riley O'Neill University of St. Thomas
Advisor(s): Cheri Shakiban, University of St. Thomas

Manual bone fragment refitting is a tedious and time-consuming task, but is critical to understanding the behavior of ancient hominids. In an attempt to digitalize and automate this process, numerous discrete geometric invariants have been prepared for use in fragment face matching and evaluated for effectiveness. Working from CT scans of bones broken by known agents (humans, hyenas, and simulated cave collapses), triangulated bone surfaces have been devised for numerical implementation in MATLAB. Four invariants have been examined for face matching: cumulative distance histograms, surface area histograms, a new method for the spherical volume invariant, and the principal curvature via Principal Component Analysis. As the meshes only serve as estimates of the surface, the optimal effectiveness follows from maximizing the accuracy of the measure and minimizing the computation time. Thus far, the volume invariant and principal curvature are the most effective measures of the four, and will be further refined and implemented for refitting.

115. Recognizing mesh structures in images

Ruoyuan Liu Carnegie Mellon University
Advisor(s): Dejan Slepcev, Carnegie Mellon University

In this project we developed methods in an image processing problem: recognizing mesh structures in images. Given a set of grayscale images representing the slices of a sample, the task is to precisely reconstruct the deformed mesh. The difficulties of this task lie in the fact that the deformations can be large, the noise is substantial, and the mesh can be damaged at places. An important part of the task is landmark detection, which aims to identify the cells and represent each cell with a single vertex. To produce reliable landmark points, low level processing such as denoising and normalization of brightness is necessary. Detection of landmark points can be done by blurring the image first and

finding all local minima points, but to precisely find landmark points in all parts of the mesh image, we are able to consider different levels of blurring in different parts of the image. The landmark detection is fundamental to further steps of recognizing mesh structures such as elastic mesh registration, which rewards agreement with the detected landmarks and the smoothness of the mesh. Additionally, we were able to build a 3D reconstruction for stacked images.

116. Internet Advertisement Improved by Machine Learning

Stephanie Skelly Western Connecticut State University
Mackensie King Western Connecticut State University
Advisor(s): Xiaodi Wang, Western Connecticut State University

Internet advertisement is the most popular form of advertisement today. Advertisement companies use this approach by gathering data from the user to show advertisements personalized by the users web history. This is composed of search history, recently visited websites and more. Some well known companies in this area are Google and Facebook, as well as others that are similar. The advertisements seen by the user are accurate, meaning the user is more likely to purchase the product being advertised to them. The purpose of this research project is to analyze the effectiveness of some of these methods, specifically the Nave Bayes and Support Vector Machine (SVM) algorithms. We will compare the effectiveness of these two methods to show that SVM is more accurate in advertisement detection.

117. Modeling and Design of Adsorption Based Filters: Bio-remediation of Heavy Metal Contaminated Water

Elvin Shoyfer Borough of Manhattan Community College CUNY
Min Shin Khant Borough of Manhattan Community College CUNY
Advisor(s): Chris McCarthy, Borough of Manhattan Community College CUNY

Our research group has developed kinetic models of adsorption-based filters. The mathematical models have been developed in support of our colleges interdisciplinary lab group (chemistry, engineering, mathematics). The lab conducts research on the bio-remediation of heavy metal contaminated water via filtration. The filters are constructed out of biomass, such as spent tea leaves. The spent tea leaves are available in large quantities as a result of the industrial production of tea beverages and in smaller quantities whenever individuals brew tea. The heavy metals bond with the surfaces of the tea leaves (adsorption). The models we have developed involve differential equations and stochastic methods. We have tested our models predictions using data from simulations and lab experiments. We have used these models to design filters which can be used to treat heavy metal contaminated drinking water.

118. Intervening in Clostridium difficile Infections in California Hospitals

Katherine Rodriguez Kean University
Advisor(s): Daniel Sewell, University of Iowa

Clostridium difficile infection (CDI) poses a serious health threat for hospitalized patients. Despite several clinical trials testing various strategies for reducing CDI rates, most have been unsuccessful or provided short term success. We used a linear mixed effects model to predict CDI rates 24 months out. Using this prediction model we tested strategies for selecting healthcare facilities to participate in a clinical trial. Our goal was to find an intervention method that effectively reduced CDI cases given an arbitrary cost constraint. Strategies under consideration included targeting hospitals with high CDI rates, larger proportions of patients over 65, and high levels of centrality within the hospital network. Our results suggest that strategic targeting of certain hospitals within a network to participate in a treatment intervention program can be beneficial to decreasing CDI cases. Targeting hospitals which have the highest incidence of CDI performed best among all tested strategies. This study provides strong evidence that convenience samples of hospitals to treat is an inefficient approach to implementing a clinical trial.

119. An Analysis of Basis-Splines and Their Applications in Treasury Yield Curves

Amaury Minino Florida Atlantic University
Sher Chhetri Florida Atlantic University
Advisor(s): Hongwei Long, Florida Atlantic University

Yield curves are one of the economic indicators routinely followed by the US Federal Reserve. Yield curves are based off the interest rates of the given bond, and their shapes tend to adjust given investor expectations. In the case of US Treasury bonds, their shape reflects the markets expectation of future economic growth. Economists and mathematicians have worked for decades to create more reliable models to forecast yield curves, given their association with recessions. As the models used by economists continue to improve, government agencies such as the Federal Reserve will have more information to enact necessary economic policies. The aim of this work is to further analyze basis splines (B-splines) as they relate to Treasury yield curves. In the last 20 years, the popular B-Spline model was extended by other researchers as they attempted to find optimal knot points. We have focused our research on optimizing the knot points used in the B-Spline model. This optimized model is then compared to other recently proposed models using US Treasury bond yields data from 1995–2015.

120. Characterizations of string stability of interconnected automobile systems

Matthew Rose Roger Williams University
Advisor(s): Hasala Senpathy Gallolu Kankanamalage, Roger Williams University

String stability plays an important role in modeling self-driving automobile systems and automated smart traffic flow systems. This plays an important role specially in designing Adaptive Cruise Control (ACC) systems and Cooperative Adaptive Cruise Control (CACC) systems. In this work we present a few variants of string stability conditions and we analyze these variants. We provide characterizations for certain types of string stability. We analyze theoretical significance of these stability notions together with numerical validations.

121. A Diffusion Maps Approach to Dimensionality Reduction

Aneesh Malhotra George Mason University
Orton Babb George Mason University
Advisor(s): Tyrus Berry, George Mason University

In topological data analysis we think of data points as having been sampled from a manifold. We used diffusion maps to approximate the Laplace-Beltrami Operator of the underlying manifold, and thus approximate the eigenfunctions of the Laplacian. Using a variety of numerical methods we attempted to use these eigenfunctions as a basis for isometrically reconstructing the data in a lower dimension

122. A Bayesian method for locating breakpoints in time series

Amy Pitts Marist College
Kathryn Haglich Lafayette College
Sarah Neitzel Unity College
Advisor(s): Jeffrey Liebner, Lafayette College

Our project proposes a new approach to finding the quantity and location of breakpoints, or change points, in time series data. This allows for more appropriate data modeling by accounting for structural changes. Bayesian Adaptive Auto-Regression (BAAR) is a Bayesian technique that samples from the distribution of number and locations of possible breakpoints. It proposes new sets of breakpoints as determined by a reversible-jump Markov Chain Monte Carlo and evaluates the proposals using the Metropolis-Hastings algorithm. Simulation results have shown that our method is able to detect changes in models, and we have provided a demonstration of BAAR as applied to the population of Pacific brown pelicans. This research was conducted at the Lafayette College REU summer program under NSF grant number 1560222.

123. Fitting rectangles under vulnerability curves: Optimal water flow through plants

Jeffae Schroff University of Houston-Downtown
Advisor(s): Koshkin Sergiy, University of Houston-Downtown

We study an optimization problem for a model of steadystate water transport through plants that maximizes water flow subject to the constraints on hydraulic conductance due to vulnerability to embolism (air blockage of conduits). The model has an elementary geometric interpretation and exhibits bottleneck behavior where one of the plant segments limits the overall optimal flow, sometimes in a counter-intuitive way. The results show good agreement with experimental measurements and provide support for the hypothesis that leaves serve as a safety buffer protecting stems against excessive embolism. This work was done at University of Houston-Downtown.

124. Investigating Cellular-Level Effects of Neurostimulation Therapies with a Partial Differential Equation Based Mathematical Model

Kaia Lindberg Roger Williams University

Advisor(s): Edward Dougherty, Roger Williams University

Neurostimulation therapies have demonstrated success in mitigating symptoms of neurodegenerative diseases, but the cellular-level impacts of these treatments remain elusive. We have implemented a mathematical model that integrates the Poisson-Nernst-Planck system of PDEs and Hodgkin-Huxley based ODEs to model the effects of this neurotherapy on transmembrane voltage, ion channel gating, and ionic mobility. The PDEs are solved using the finite element method on a biologically inspired discretized domain. Our results suggest two possible mechanisms by which neurostimulation achieves therapeutic success. First, neurostimulation polarizes the cell membrane, elevating resting membrane potential to facilitate action potential firing. Second, a neurostimulation-induced calcium influx alters cytosolic calcium concentrations, which is essential for proper neurotransmitter secretion and its dyshomeostatis is a known associate of neurodegenerative diseases. We also compare the effects of two different types of neurostimulation (transcranial electrical stimulation and deep brain stimulation) showcasing cellular-level differences resulting from these distinct forms of electrical therapy.

125. A Computational Approach for Constructing an Intracellular Signaling Pathway Mathematical Model with Application to Parkinson's Disease

Elizabeth Gilchrist Roger Williams University

Abigail Small Roger Williams University

Advisor(s): Edward Dougherty, Roger Williams University

Parkinsons disease (PD) is the second most common neurodegenerative disorder. Despite this, there is no cure and the cellular level pathogenesis remains elusive. In an attempt to gain new insights, we created a mathematical model of the intracellular signaling pathway of a dopaminergic neuron cell with application to PD. A comprehensive literature search was conducted to construct a wiring diagram, which was used to generate a system of ordinary differential equations using the law of mass action and the Michaelis-Menten equation. Many of the kinetics are presently unknown, so a novel computationally-based reverse engineering method was used to identify them; this approach uses expected system behavior and the Metropolis Algorithm to numerically determine appropriate values. Suitable rates were ranked based on performance in a phenotype-based computational assessment, and then robustly screened using a k-means clustering assessment, sensitivity analyses, and an eigen-analysis. The result is a mathematical model that efficiently emulates the signaling network of a dopaminergic neuron model; it showcases the intracellular processes of both a healthy and PD-like dopaminergic neuron.

126. A Mathematical Approach for Assessing tDCS Efficacy for Post-Traumatic Stress Disorder

Abigail Small Roger Williams University

Advisor(s): Edward Dougherty, Roger Williams University

Post-Traumatic Stress Disorder (PTSD) is a neurological condition caused by distressing or traumatic events. It has been recently found that symptoms of PTSD can be combated using forms of neurostimulation, in particular, transcranial direct current stimulation (tDCS). While it is known that the electrical energy delivered by this treatment to targeted areas of the brain is effective in treating PTSD, the optimal positioning of tDCS electrodes and treatment parameters for achieving the greatest efficacy is unknown. We have implemented a partial differential equation based mathematical model of tDCS with application to PTDS, and have generated numerous numerical simulations using the finite element method, all using distinct electrode montages, treatment parameters known to mitigate PTSD symptoms, and a three-dimensional MRI-derived cranial cavity with biologically-based tissue conductivities. The model predicts not only voltage and electrical current density within the head cavity, but also the sensitivity of the brain tissue to fire an action potential during treatments. We present our current results and findings that begin to shed light on ideal tDCS settings for treating PTSD.

127. A Validation Method for Point-Count Surveys

Christopher Kromm California Lutheran University
Advisor(s): Christopher Brown, California Lutheran University

Models for monitoring the counts of birds range from route regression to Bayesian hierarchical models. This study presents an alternative three-step model framework to predict the number of birds counted by species in a region. The model framework 1) imputes missing data, 2) splits the species into multiple forecast methods based on variance, and 3) forecasts bird counts using a forecast method based on the split. Three imputation methods (Zero, Mean, and Probability-of-Detection) were compared using USGS Breeding Bird Survey data. The species were split using a power function with a retrospective residual analysis. Species with higher variance counts where predicted using a Poisson or Negative Binomial distribution (based on the dispersion parameter) centered on the mean of the modified moving average of the hindcast year counts. Species with lower variance counts where forecasted with the mean of the modified moving average of the hindcast year counts. The zero-imputed model best predicted the number of birds counted with an absolute residual score of 105758. These results indicate this framework reasonably predicts counts allowing for a new method to validate bird surveys.

128. A Game-Theoretical Approach to Middle East Respiratory Syndrome

Kenyona Bethea Bennett College
Advisor(s): Ajanta Roy and Igor Erovenko, Bennett College and University of North Carolina at Greensboro

Middle East Respiratory Syndrome (MERS) is a global threat and has the potential to cause large outbreaks with substantial public health and economic costs. It is caused by a single-stranded RNA virus, called a coronavirus. The virus is transmitted from human-to-human and one of the most effective strategies to prevent the disease is usage of protective gear (i.e., masks and gloves) in nosocomial settings. With no vaccination or cure, MERS has become a growing concern. In this project, we present a game-theoretical model in which individuals choose to either use protective gear or not. We used MATLAB to plots results of our project. This work was done as a part of a NREUP project.

129. Is Condom Usage Beneficial to Maximize Protection against Chlamdyia

Carolina Herrera UNCG
Ciera Tucker UNCG
Advisor(s): Ajanta Roy and Igor Erovenko, Bennnett College and UNCG

Chlamydia is an infectious disease spread by the bacteria Chlamydia Trachomatis that affects millions of people worldwide. The disease is spread through sexual contact between an infected and susceptible individual. Protective measures include abstinence and long-term monogamy, but the most common and simple method is condom usage. In this project, we took a game theoretic approach to figuring out the optimal protection level needed to eradicate Chlamydia with condom usage as our protective measure of choice. We performed a sensitivity analysis to demonstrate the effects of changing parameter values on the optimal solution of the system.

130. Effects of observation function selection in nonlinear filtering for epidemic models

Leah Mitchell Worcester Polytechnic Institute
Advisor(s): Andrea Arnold, Worcester Polytechnic Institute

Nonlinear filtering is an approach to solving the inverse problem of estimating unknown states and/or parameters of a system. The ensemble Kalman filter (EnKF) is one such algorithm that can be used for nonlinear, non-Gaussian systems within a Bayesian inference framework. One component of the EnKF is the observation model, which relates the discrete, noisy data back to the system model. The observation model can take different forms based on assumptions relating to the available data and relevant system parameters. The goal of this research is to explore the effects of selecting different observation models in the EnKF framework. In particular, four different observation models, of different forms and various levels of complexity, are examined through an application to epidemiology. Results discuss the effects of the observation model selection on the filter output.

131. Theoretical Notions of Ecological Stability and Their Relation to Temporal Variability

Chace Covington Francis Marion University
Advisor(s): Craig Jackson, Ohio Wesleyan University

Ecological stability describes how populations of species in an ecosystem behave after a disturbance and can be measured empirically and theoretically. Our study uses a first-order multivariate autoregressive model framework to explore possible relationships between empirical and theoretical measures of stability and possible relationships between different theoretical measures of stability. The empirical measures of stability included in this study are three coefficients of variation (CV): the average population CV, the weighted average population CV, and the community CV. All theoretical measures of stability included are derived from a theoretical community matrix and include asymptotic resilience, initial resilience, and intrinsic stochastic invariability. We find no evidence for any relationship between empirical and theoretical measures of stability, agreeing with previous experimental research by Downing et al. We observe clear relationships between different theoretical measures of stability. We formalize these relationships with inequalities similar to those derived by Arnoldi et al. for continuous models. This study was done at Ohio Wesleyan University's REU program.

132. Behavioral Synchrony and Functional EEG Networks

Sarah McGuire College of the Holy Cross
Advisor(s): David Damiano, College of the Holy Cross

People engaged in social interactions exhibit natural, unintentional coordination of their body movements. Although intense efforts have been made to localize behaviors in human brain activity, little is known about the functional networks that underlie human social interaction. We are interested in how social behavioral synchronization corresponds to functional networks in the brain. To this end, we analyzed electroencephalograph (EEG) activity from an experiment involving participant pairs swinging pendulums in different interpersonal coordination conditions. Using methods of computational topology and network theory, we are able to identify functional networks based on pair-wise coordination between electrodes. Initial results indicate the existence of dynamic network features across different coordination conditions and frequency bands. In future work, we will apply these novel network analysis techniques to evaluate the social brain networks in adolescents with Autism Spectrum Disorder (ASD), as one of the suggested tendencies of people with ASD is social disconnection.

133. Analyzing Behavior and Brain Activity Quantitatively in Mice

Chanel Fraikin University of California, San Diego
Katrina Gensterblum Michigan State University
Advisor(s): Mark Reimers, Michigan State University

It is well known that the brain controls behavior. However, so far we have little idea about the relationship between brain activity and spontaneous motion. In order to characterize how specific neural patterns relate to actions, we need a better taxonomy of the almost infinite variety of observed behaviors. First, we prepared hand-coded annotations from recordings of mouse activity to train a machine learning algorithm that will automatically extend these annotations to other videos using an artificial neural network. Second, we used large-scale recordings over several areas of the brain taken simultaneously with the behavior to identify neurons whose activity is correlated with particular aspects of spontaneous behavior. We gratefully acknowledge support from the National Security Agency (NSA Award No. H98230-18-1-0042), the National Science Foundation (NSF Award No. 1559776), and Michigan State University.

134. Tracking Neural Activity: Automated Image Analysis

Amanda Stanley Grand Valley State University
BethAnna Jones State University of New York at Geneseo
Advisor(s): Mark Reimers, Michigan State University

Modern optical methods such as high-definition two-photon imaging of calcium fluorescence allow us to view the activity of tens of thousands of neurons in the brain. Due to the large number of images to sort through, manual methods of location, identification, and analysis of these individual neurons is tedious and time-consuming. Researchers need an automated system to complete in hours what may take an expert weeks or years. Some automated techniques exist, but as shown by the 2017 SURIEM project, they find only roughly half the cells, and disagree on half of what they find. We aim to build on this work and improve automatic analysis techniques of neural images. Current leading methods

analyze neural images using singular value decomposition and constrained non-negative matrix factorizations. We will compare the cells identified with these methods and improve them by i) analyzing fluorescence patterns of the cells; ii) investigating correlation patterns across cells; and iii) aligning cells between sessions to compare cell activity across days as animals learn. This work was completed as part of the 2018 SURIEM program supported by Michigan State University, the NSA, and NSF.

135. Fuzzy Game Theoretic Approach — Hand Foot Mouth Disease

Hannah Ross Bennett College
Kristina Smith Bennett College
Advisor(s): Ajanta Roy, Bennett College

In this project, we present a fuzzy game theoretic approach to Hand-Foot-Mouth disease. Hand-Foot-Mouth-Disease (HFMD) is an acute contagious disease, one that has affected many populations around the world. It is caused by viruses belonging to the enterovirus genus (group). The group of viruses includes polio viruses, coxsackie viruses, echo viruses, and entero viruses. Although there are several viruses, coxsackie virus A16 and enteroviruses 71 (EV71) are the most common to produce HFMD. Precise estimate of disease transmission rate is critical for epidemiological Game theory model. We considered the transmission co-efficient to be uncertain and described by a (symmetric) triangular fuzzy number. We defuzzified the optimum results by a signed distance method. We studied sensitivity analysis to demonstrate the effects of changing parameter values on the optimal solution of the system.

136. Oyster population dynamics: a stage-structured differential equation model of interacting reefs

Rachel Wilson The College of William and Mary
Advisor(s): Leah Shaw, The College of William and Mary

Oysters have biological importance due to their filtration and habitat building abilities, aside from their commercial importance. Oysters begin as free-swimming larvae that attach to a substrate (oyster shells) and mature from juveniles to adults. Adult oysters produce larvae to replenish the population. Our goal is to model oysters using differential equations to understand the interactions between populations. A stage-structured model was developed with the equations representing juvenile oysters, adult oysters, dead shell, and sediment deposition. Larval sources were existing adults and a background larval availability parameter. One-population and two-population models were developed. Bifurcation diagrams were analyzed to determine the stability of positive states while varying the larval sources and population interactions (ex: source/sink dynamics). Critical reef height was also analyzed this determines the height the reef needs to be initially in order to persist over time. This analysis informs the building of new reefs for oyster restoration, showing the benefits of using adult oysters as well as dead shell. Initial funding from the EXTREEMS-QED REU at William & Mary.

137. Asymmetric Demographic Models with a Mate-Finding Allee Effect

Jared Ott University of Nebraska - Lincoln
Elizabeth Anderson Villanova University
Gwyneth Terrett Taylor University
Advisor(s): Daniel Maxin, Valparaiso University

In a two-sex demographic model, the most challenging mathematical components are the couple-formation functions. These functions link the number of pairs with the number of available singles. They are usually not detailed enough to include important aspects of social behavior such as: motivation for pairing which may be gender specific, scarcity or abundance of the opposite gender or social/economic factors. In this research we analyze several two-sex models to better describe asymmetric demographic situations. In particular we focus on a mate-finding Allee effect which models the difficulty of pairing at low population densities and investigate whether this effect is sensitive to changes in sex ratios and/or overall female/male densities. We also compute the Allee threshold which separates population extinction from persistence and test these results against real demographic data from world populations. This project was completed at an REU Program hosted by Valparaiso University.

138. Analyzing the Dynamics of an Inflammatory Response to a Bacterial Infection in Rats

Allison Torsey SUNY College at Buffalo
Advisor(s): Julia Arciero, Indiana University Purdue University Indianapolis

Sepsis is a serious health condition defined by an overactive immune response that causes severe damage to healthy tissue, often resulting in death. Mathematical modeling has emerged as a useful tool to investigate key elements of the immune response and thus offers a useful method for studying sepsis. Here, a system of four ordinary differential equations is developed to simulate the dynamics of bacteria, the pro-inflammatory immune response, anti-inflammatory immune response, and tissue damage. Model parameters are fit to experimental data from rat sepsis studies. The model is used to predict the survivability range for an infection while varying the initial amount, growth rate, or virulence of the bacteria in the system. For highly virulent strains of bacteria, aseptic or septic death is predicted for very small levels of initial bacterial loads. Model predictions are also used to explain the experimentally observed variability in the mortality rates among rats. This work was supported by the NSF through an REU program at the Mathematical Biosciences Institute at Ohio State University and Indiana University Purdue University Indianapolis.

139. Variations of the multistage model of carcinogenesis

Grant Clark Adelphi University
Brian Seidl Adelphi University
Scott Shannon Adelphi University
Advisor(s): Josh Hiller, Adelphi University

Multistage models of carcinogenesis have been a cornerstone of mathematical oncology since they were first introduced in the 1950s and 1960s. In this project, we explore several important variations of these models that appear in the epidemiological literature and we examine their ability to predict, and fit, real world cancer incidence data.

140. Effects of maternal complications on circulating protein profiles in pre-term infants

Samantha Bothwell Colorado State University
Advisor(s): Brandie Wagner, University of Colorado Anshutz

Infants born preterm (24–34 weeks post menstrual age) are at high risk for developing chronic lung disease of prematurity, also known as bronchopulmonary dysplasia (BPD), which is characterized by prolonged need for oxygen therapy, frequent pulmonary infections requiring hospitalizations, asthma, exercise intolerance, and pulmonary hypertension. BPD results from the adverse effects of early exposure to the extrauterine environment and the need for supportive interventions such as mechanical ventilation and high levels of oxygen delivery. These stimuli lead to inflammation of the immature lung and disrupt normal lung development. Recently, maternal complications resulting in premature birth have been associated with different inflammatory profiles. A better understanding of the mechanism of the risk associated with maternal complications and inflammation in the infant lung is needed. This project will use a high-dimensional proteomic platform ($\sim$1,100 analytes) to determine the association between maternal complications during pregnancy and protein profiles in the circulating blood of pre-term infants at risk for BPD.

141. Mutualistic Model for Coffee-Bee Interaction for Farm Recovery

Erick Orengo Inter-American University of Puerto Rico, Bayamn Campus
Advisor(s): Carmen Caiseda, Inter-American University of Puerto Rico, Bayamn Campus

Extreme weather conditions, such as hurricane Maria that hit Puerto Rico in 2017, devastate farms requiring great unified efforts toward recovery. In this project we study the productive mutualistic relation between bees and coffee with the hope of contributing knowledge and tools for farmers to rebuild agriculture. We have developed a facultative unidirectional mathematical model, analyzed stability and estimated optimal parameters. We have also designed and developed an application tool for farmers to compute the necessary relation between the mutualistic species after a disaster, that will prevent extinction or negative exploitation of the species.

142. Mathematical models for the island fox, feral pig, golden eagle system incorporating a strong demographic Allee effect in the island fox

Caroline Sinclair California Lutheran University
Advisor(s): Christopher Brown, California Lutheran University

In a robust mathematical model of the interaction of golden eagle, feral pig, and island fox populations, we determine the potential behaviors of the model system for various parameter values. The mathematical models for the golden eagle, feral pig, and island fox system incorporates a strong demographic Allee effect in the island fox population. Two different Holling functional responses will be investigated as predation terms in the models. Under certain restricting conditions, the different models produced coexistence equilibrium points. We examine boundaries in the parameter space that describe bifurcations of the number of equilibria and their stabilities. The comparison between the restoration of the island fox and the predicted behavior from the model will better inform conservationists of the impacts that different predator and prey species have on the species in question, specifically with regards to the island fox conservation efforts.

143. A Mathematical Model of West Nile Virus: The Effect of Interaction Between Humans, Mosquitoes, and Birds

Noelle West Dixie State University

Advisor(s): Vinodh Chellamuthu, Dixie State University

West Nile virus (WNV) is a mosquito-borne virus that circulates among birds but can also affect humans and horses. Migrating birds carry these viruses from one place to another each year. West Nile virus (WNV) has spread rapidly across the continent resulting in numerous human infections and deaths. Several studies suggest that larval mosquito control measures should be taken as early as possible in a season to control the mosquito population size. Also, adult mosquito control measures are necessary to prevent the transmission of WNV from mosquitoes to birds and humans. To better understand the effective strategy for controlling affected larvae mosquito population, we have developed a mathematical model using a system of first order differential equations to investigate the transmission dynamics of WNV in a mosquito-bird-human community. We also incorporated local temperature data in the model to show the impact of birth and growth rates of the mosquitoes and the migration patterns of birds in disease dynamics. Our model could be used by mosquito abatement centers to determine optimal strategies to efficiently control disease outbreaks.

144. Computing Fixation Probabilities under the Death-Birth process on Random Graphs

Lori Brizuela Emmanuel College

Joshua Kolodny Emmanuel College

Advisor(s): Christine Sample, Emmanuel College

In the past few decades there has been an emergence of interest related to evolutionary dynamics on graphs. Using the death-Birth process, we studied the fixation probability of mutants in different graph structures. We developed novel techniques to evaluate the effect of a graph structure on a mutant's success for weak selection—meaning that a mutant's fitness is close to that of the resident type. Based on the fixation probabilities, we classified graphs into three categories: a suppressor, which limits an advantageous mutant's ability to fixate; an amplifier, which promotes the ability to fixate; and a neutral graph, which has no effect. We derived an approximation for the success of a mutation based on the first and second moments of the degree distribution. We numerically analyzed Erdős-Renyi and Barabasi-Albert graphs and we found that a higher average degree leads to a weaker suppressor and that a larger graph leads to a stronger suppressor. The derived approximation is highly accurate for both random graph models.

145. Ecological Niche Modeling and Risk Assessment of Thousand Cankers Disease

Benjamin Reber Houghton College

Brianna Alred University of Tennessee, Knoxville

Advisor(s): Mona Papes, University of Tennessee, Knoxville

Thousand Cankers Disease (TCD) is an arboreal disease complex which causes mortality in eastern black walnut trees (Juglans nigra). TCD is caused by the fungus Geosmithia morbida, which is vectored by the walnut twig beetle (Pityophthorus juglandis). Maxent models were created to predict niche shift of P. juglandis and J. nigra under conditions predicted by multiple GCMs using RCP 4.5 and RCP 8.5 in order to help determine high risk areas for TCD. The niche of P. juglandis was predicted to have a wide extent which was largely plastic to future scenarios. A northward shift was predicted for the niche of J. nigra in future scenarios. The highest risk areas for TCD were predicted to be in the southern U.S. due to more stressful conditions on the trees. Research was done as a part of summer research program at the National Institute for Mathematical and Biological Synthesis at the University of Tennessee, Knoxville.

146. Modeling Networks of Evolving Populations

Sean Elliott MIT PRIMES
Advisor(s): Dominic Skinner, MIT

In this work, we build a mathematical model for evolution based on the Fisher-Eigen (FE) process. The FE partial differential equation describes the evolution of a probability density function representing the distribution of a population over a phenotype space. This equation depends on the choice of a fitness function representing the likelihood of survival at each point in the space. The FE model has been studied analytically for simple fitness functions, but in general becomes too complex to analyze or simulate when many phenotypes are included. Our goal is to form a model that captures the FE behavior but enables the inclusion of morepotentially 1000sof phenotypes. For our design, we first simulate the FE process over a 2D fitness landscape using MATLAB. We assume that organisms can be clustered into groups that share similar genes, and we create a system of ordinary differential equations over the network of these groups. Finally, we show that our model finds the correct equilibrium solution. The result is a computationally feasible model that captures the behavior of a population over time, allowing researchers to predict dynamic behavior based on equilibrium data.

147. Modeling and Predicting the Dispersal of Lycorma delicatula in North America

Jessica Shi Jamestown High School
Advisor(s): Junping Shi, College of William & Mary

The invasive spotted lanternfly, Lycorma delicatula, was introduced to Berks County Pennsylvania in September 2014. Since then, it has spread to over 4.5 million acres in Pennsylvania as well as to locations in other states in the eastern United States. We used a diffusion equation to model the spread of the species and predict its growth if no control mechanism is implemented. The invasion rate was estimated using available data of infected areas and statistical tools.

148. Amplifiers of Selection for the death-Birth Process

Patricia Steinhagen Emmanuel College
Robert Jencks Emmanuel College
Advisor(s): Benjamin Allen, Emmanuel College

The fixation probability of a mutation depends on the spatial structure of the population. Using the death-Birth process on graphs, we mathematically analyzed the effect of different population structures under both weak and non-weak selection. We analytically determined fixation probabilities on three different graph structures as a function of mutant fitness. These graphs were classified by comparing their fixation probability to a well-mixed population of equal size. We found population structures that suppress selection and others that reduce fixation universally. We also discovered the first known structure capable of amplifying selection for moderately beneficial mutations.

149. Partial Least Squares Analysis of fMRI Brain Scans

Kaila DeChristofaro Slippery Rock University
Jessica Lefler Slippery Rock University
Rebecca Himes Slippery Rock University
Advisor(s): Dil Singhabahu, Slippery Rock University

The purpose of this project is to analyze fMRI brain scans of children whose mothers with substance abuse histories during pregnancy and determine if there is a difference in the attention of children of the mothers who took substances compared to those who did not. The data being used is from the Maternal Health Project at the University of Pittsburgh, and is divided into three groups: one control group and two test groups, each of size fifteen. The brain images were taken 21 years after the children were born. The goal is to determine if there is a difference in attention networks of each group. Currently, the method being used to model this is linear regression model, but this specific project is focused on using the partial least squares method. We will explore the mathematics behind PLS before using it on the fMRIs. The reason we chose this method is because the data is highly correlated, and there is a very small sample size. The partial least squares method can account for this. Furthermore, current modeling has found no statistically relevant difference in the attention of children between the three groups. We believe that a difference can be found when using PLS.

150. Optimal Control and Longevity of Treatment in a Seasonal Model of Swimmer's Itch

Jordan Pellett University of Wisconsin- La Crosse
Advisor(s): James Peirce, University of Wisconsin- La Crosse

Swimmer's itch is an emerging disease caused by flatworm parasites that use water birds as definitive hosts and aquatic snails as intermediate hosts. When parasite larvae mistakenly penetrate human skin, they initiate an inflammatory skin reaction leading to intense itching and discomfort. While swimmer's itch has plagued Midwestern lakes for decades, a recent rise in the global occurrence and its subsequent impact on recreational activities has increased the interest in implementing effective control regimes. However, optimal control strategies may change depending on the costs associated with treatment and the targeted reduction in human swimmer's itch cases. We have derived a sequence describing the prevalence of infected bird hosts in the years after treatment. The sequence increases asymptotically towards the initial prevalence providing a method for recommending the time to the next optimal treatment. Here, we share the results of our investigation into the longevity of treatment when various optimal control strategies are applied to a mathematical model of swimmer's itch.

151. Modeling Transmission Patterns of White-Nose Syndrome in Little Brown Bats

Claire Wyble Western Connecticut State University
Karim Naba Western Connecticut State University
Advisor(s): Xiaodi Wang, Western Connecticut State University

White-Nose Syndrome (WNS) is a pathogenic disease caused by the fungus Pseudogymnoascus destructans (Pd). Since the first recorded case in 2006, WNS has killed millions of North American bats. Originating near Albany, New York, WNS has since spread into Canada as well as along the East Coast of the United States and has been reported as far inland as Kansas. As of 2015 one case of WNS has been confirmed in northwest Washington, and as of now is the only recorded incidence on the West Coast. Although WNS affects a wide range of species, we focus solely on the little brown bat (Myotis lucifugus) both because of its high susceptibility to contract WNS as well as its prevalence across North America. Using data collected on East Coast little brown bat populations we construct a distance-based diffusion model to depict the patterns of transmission. Utilizing machine learning algorithms and Monte Carlo simulations, we then adapt our model to predict the anticipated spread of WNS along the West Coast.

152. Using Global Sensitivity Analysis to Find Influential Parameters in a Wound-Healing Model

Abdullah Ateyeh Western Kentucky University
Rithik Reddy Western Kentucky University
Advisor(s): Richard Schugart, Western Kentucky University

To formulate a mathematical model that accurately represents the physiology of a wound, the model must easily predict the most influential factors that affect the wound-healing process. Using a differential-equation model that describes the interactions among matrix metalloproteinases, their inhibitors, the extracellular matrix, and fibroblasts (Krishna et al., 2015), this work focuses on two approaches using global sensitivity analyses. In the first approach, two matrices are constructed and then filled with quasi-random numbers chosen from a specified uniform distribution. From this, Sobol or sensitivity indices are computed for each patient, and then results are evaluated. The next method is Morris screening, which measures the change in the state variables when a specific parameter is slightly modified from the predicted value. A sum of squares of the differences between the old and modified model can be used to give the overall influence each parameter has on the model. Overall, these methods have help us find the most significant factors in the wound-healing process, which can further be used to more accurately predict the healing process for individual patients.

153. Modeling of the Growth of *Chlorella vulgaris* with Respect to Manganese Dosage

Annabella Pauley Marshall University
Advisor(s): Anna Mummert, Marshall University

With the expansion of industrialization, the ramifications of increased pollution in the environment are critical to understand. *Chlorella vulgaris*, a unicellular green alga, was subjected to various concentrations of manganese, a potential pollutant which is found in batteries and mine effluent water. The changes in growth due to manganese

dosage were measured by the levels of various metabolites as well as the optical density of the cultures over time. A logistic growth pattern is seen with all dosages of manganese, though the trend changes with increased dosage. A modified logistic model that accounts for the variations due to increased dosing was fit to the data collected. The modifications to the logistic model were influenced by insight into the biological processes taking place. Because of this, the model has the potential to give information about the relationship between these biological processes, conveying a more comprehensive understanding of the effect that manganese has on *Chlorella vulgaris*.

154. Is a Split Gender Mathematical Model Useful in Modeling Zika?

Matthew Bush Youngstown State University
Advisor(s): Alicia Prieto, Youngstown State University

Mathematical modeling has, for a long time, played an important role in predicting and identifying potential epidemiological outbreaks. Due to recent world wide outbreaks, Zika virus is one vector borne disease garnering a lot of attention. Recent models of Zika have taken into account things such as sexual transmission and even symptomatic versus non-symptomatic transmission. With our model we seek to explore whether it is necessary to split the model not only by symptomatic and non-symptomatic, but also by gender. The goal of this model is to find the basic reproductive number individually in each gender and together to determine whether a split gender model is necessary in modeling Zika.

155. An Agent Based Model for the Dynamics of HPV with the Integration of Vaccination

Stefano Chiaradonna Benedictine University
Advisor(s): Timothy Comar, Benedictine University

The Human papillomavirus (HPV) is one of the most prevalent sexually transmitted diseases in the United States with over 79 million individuals currently infected. In order to investigate the spread of HPV, we propose and analyze an Agent Based Model (ABM) via Netlogo that encompasses age, sexual activity, and heterosexual interactions. In addition, the model integrates the recent CDC recommendation of having adolescents of ages 11 and 12 receive only two doses rather than the full three dose vaccine regimen. The ABM, due to its stochasticity, allows us to investigate a variety of scenarios such as changing demographics and observe how the infection spreads for each scenario. From these scenarios, we amalgamate the data and find an optimized, vaccination strategy that targets particular age groups.

156. Hypothesizing Directionally Dependent Neurons through a Computational Model of the Primary Visual Cortex

Harrison Tuckman College of William and Mary
Advisor(s): Mainak Patel, College of William and Mary

The primary visual cortex (V1) is the region of the brain responsible for processing visual information. This region of the brain is divided into hypercolumns which send information from V1 to the middle temporal visual area (V5) of the brain, which contains directionally dependent neurons. These neurons use information from V1 to determine motion in the visual field. The goal of this research is to hypothesize the mechanism for these cells through a computational model of the V1 and the V5. We began our model with the integrate and fire model for interacting neurons, derived from the Hodgkin-Huxley model. After modeling one neuron, a network of neurons composing a single hypercolumn was created. Once the hypercolumn was created, it was tested for realistic behavior through empirically supported tests, such as the contrast reversal test. After ensuring their realistic behavior, multiple hypercolumns were connected to the V5. Different arrangements of neurons in the V5 were then tested for the emergence of directionally dependent neurons. Early results show that the coupling of fast firing neurons with slow firing neurons may lead to the emergence of directional dependent behavior.

157. Controlling Foodbourne Infections in Lettuce: Testing and Cleaning Methods for Curbing the Spread of *E.* coli O157:H7

Emily Dorn Olivet College
Advisor(s): Baojun Song, Montclair State University

In the US, 96,534 humans are infected each year by the Shiga toxin-producing *E. coli* O157:H7. In 2018, lettuce farms in the Yuma Region contracted the bacteria *E. coli*, which is a recurring problem in foodborne disease outbreaks. In

this study, we propose a new mathematical framework to capture the dynamics of the spread of *E. coli* infection in lettuce due to contaminated soil and contaminated equipment used in infection-free fields without proper treatment or cleaning. The mathematical modeling approach in the study is to designate farming equipment as the vector of *E. coli* infection and lettuce as the host. Farm equipment is also essential to the growing process of lettuce. The results of the study show that by cleaning farm equipment the yield of healthy lettuce increases. This research was conducted with the Mathematical and Theoretical Biology Institute at Arizona State University.

158. Testing the Performance of the MutPred2 Variant Pathogenicity Predictor on a set of BRCA1 and BRCA2 Variants and the Relevance of Solvent Accessibility in Predicting Pathogenicity.

Diego Quezada-Munoz California State University, Monterey Bay

Advisor(s): Melissa Cline, University of California, Santa Cruz

Mutations in the BRCA1 and BRCA2 genes are known to be associated with an increased risk of breast and ovarian cancer. Variants of BRCA1 and BRCA2 genes are clinically assessed to be classified as pathogenic(causing-cancer) or benign. Still there are variants that have not been clinically assessed and are known as Variants of Uncertain Significance. Individuals who have a variant ion this kind could possibly be in risk of developing cancer. So in absence of clinical data, In Silico Prediction Algorithms are techniques that predict the probability of a variant being pathogenic by using parameters such as protein structure of the variant and its amino acid substitution(s). The purpose of this study is to test the performance of the In Silico pathogenicity prediction software known as MutPred2 and the predicted relative solvent accessibility of an amino acid. To address this question we applied the MutPred2 prediction software on a set of single-amino acid change BRCA1 and BRCA2 variants. Performance is based on how MutPred2 predictions and relative solvent accessibility predictions compare to clinically-assessed pathogenicity classifications in the ENIGMA and ClinVar database.

159. Discovering Temporal Contingency

Nathan Anderson Texas A&M University

Advisor(s): Heath Blackmon, Texas A&M University

Evolutionary biologists frequently wish to know if two traits are correlated or share some functional relationship. For example, nocturnality should, in theory, be correlated with the presence of camouflage because there is much less selective pressure on camouflage for organisms that are active in the dark and a biologist would like to confirm this hypothesis with a statistical test for correlation in the evolution of those two traits. However, most of the methods available for understanding correlation or contingency in the evolution of discrete traits are based on detecting differences in the rate of transitions in one trait when it is associated with a specific state of the other trait. These methods lead to several known problems, such as an elevated false positive rate due to a low number of transitions affecting a large large area of the tree. Transitions located early in the phylogeny, or before a number of speciation events are especially troublesome. We solve this so-called problem of within clade pseudo replication by developing an approach that focuses on temporal correlations in the transitions of discrete traits.

160. Analysis of Dynamics in Neuronal Networks

Aleksander Marczuk University of Hartford

Advisor(s): Hwayeon Ryu, University of Hartford

Using the FitzHughNagumo equations for dynamic systems, previously developed models of neuronal interactions have shown results relating the existence of coupling delays to the synchronization of a two-neuron network. By adding a third neuron to the system, MATLAB and XPP codes were used to discover new results. These findings have significant differences from prior research and show which delays are most important in creating synchronization within the system. In addition, these discoveries can provide insights on how to expand delayed differential equations representing neuronal networks to an arbitrary number of neurons.

161. Active Matter: Modeling Chemotaxis

Adama Sene Borough of Manhattan Community College CUNY
Jorwyn Medina Borough of Manhattan Community College CUNY
Muhammad Hannan Borough of Manhattan Community College CUNY
Gianni Watts Borough of Manhattan Community College CUNY
Advisor(s): Chris McCarthy, Borough of Manhattan Community College CUNY

Active matter research focuses on the principle of emergence, wherein order and structure emerge from the bottom up. The phenomenon is present in nature as organisms organize themselves without top-down commands, e.g. birds fly in flocks, fish travel in schools. Simple rules can lead to complex behavior, e.g. chemotaxis and self-assembly. We have created agent-based simulations of active matter phenomena such as chemotaxis. Chemotaxis is when the motion of an organism, such as a bacterium, is determined by an external chemical gradient. The simulations allow us to test mathematical models of the phenomena under varying conditions. We also use the simulations to gain general insights into the mathematics of emergence and agent based models.

162. A Semi-Supervised Dimensionality Reduction Method to Reduce Batch Effects in Genomic Data

Anusha Murali Bishop Brady High School
Advisor(s): Mahmoud Ghandi, Broad Institute

Gene expression datasets generated using different experimental methods suffer from batch effects. In this study, we present a novel dimensionality reduction method and apply it to reduce batch effects in the Cancer Cell Line Encyclopedia mRNA expression datasets. We formulate our investigation as a constraint optimization problem to find a projection that minimizes the normalized distances between paired samples, and solve it using Lagrange multiplier technique for a special case. We demonstrate a geometric representation of the method with a few examples. Finally, we provide a solution to the general case using an eigendecomposition technique.

163. Simplicial Complexes of Zero-Sumfree Sets

Ashleigh Adams University of Minnesota Twin Cities
Carole Hall University of Minnesota Twin Cities
Advisor(s): Kaisa Taipale, University of Minnesota Twin Cities

Additive combinatorics is the study of counting objects in the context of a natural additive operation. One major goal in this field is to find the maximum size of an ℓ-zero-sumfree subset of a general abelian group. Consider the set of all ℓ-zero-sumfree subsets of the integers modulo n; this is a simplicial complex denoted by $\Delta_{n,\ell}$. We varied the parameters n and ℓ and explored the resulting facet structures, cyclic sieving phenomena, and Whitney numbers. We determined these characteristics for two distinct classes of $\Delta_{n,\ell}$: first, when $n = 2\ell$, and second, when n is twice a prime number and $n - \ell < 4$.

164. Conjecture O holds for some Horospherical Varieties of Picard Rank 1

Lela Bones Salisbury University
Garrett Fowler Salisbury University
Advisor(s): Ryan Shifler, Salisbury University

Property O for arbitrary complex, Fano manifolds X, is a statement about the eigenvalues of the linear operator obtained from the quantum multiplication of the anticanonical class of X. Pasquier listed the non-homogenous horospherical varieties of Picard rank 1 into five classes. Property O has already been shown to hold for the odd symplectic Grass-mannian which is one class. We will show that Property O holds for two more classes and an example in a third class of Pasquiers list. The theory of Perron-Frobenius reduces our proofs to be graph theoretic.

165. Asymptotics of Visibility in *n*-Dimensional Grid Worlds

Srinivasan Sathiamurthy Lexington High School
Ezra Erives Lexington High School
Advisor(s): Zarathustra Brady, MIT

We focus on $\mathbb{N}^3$, imagined as a three dimensional, axis-aligned, grid world partitioned into $1 \times 1 \times 1$ unit cubes, each of which is considered to be empty, in which case a line of sight can pass through it, or obstructing, in which case no line of sight can pass through it. From a given position, some of these obstructing cubes block ones view of other obstructing cubes, motivating the following question: What is the largest number of obstructing cubes that can be simultaneously visible from a point, given that all of the obstructing cubes lie within a cube of fixed size? We present a model through which the problem of visibility is turned into one of partially ordered sets, yielding an $\Omega(n^{\frac{8}{3}})$ lower bound, where n is the size of our cube. The previous work along with additional analytic techniques are used to prove an $O(n^{\frac{8}{3}} \log n)$ upper bound in a reduced visibility setting. Finally, an upper bound in a general visibility context is given, as well as generalizations of the previously mentioned bounds to $d > 2$ dimensions.

166. Foldings on Generalized Bonding Graphs

Bianca Salinas St. Edward's University
Advisor(s): Mitch Phillipson, St. Edward's University

In this poster, we explore non-crossing matchings on words from the vertex set of a graph, where each arc in the matching is a corresponding edge in the graph. These matchings are related by a process called a move. We explore the number of matchings on a given word. Additionally, we define when two graphs are equivalent and classify graphs acting on sequences of length less than 8.

167. Generating Functions for f-vectors and the cd-index of Weight Polytopes

Vaughan McDonald Harvard University
Johnny Gao MIT
Advisor(s): Victor Reiner, University of Minnesota

We study the f-vector and cd-index of a weight polytope (or Wythoff polytope), which is the convex hull of the orbit of a point in space under the action of a finite reflection group. We show that a formula of Renner for the f-vector in the Weyl group case is also valid for arbitrary finite reflection groups, via results of Maxwell. We then use this to continue work begun by Golubitsky, giving generating functions for the f-vectors in all of the cases where the weight polytope is simple. We then further apply Maxwell's results to study the cd-index in non-simple cases. Namely, we give a generating function for the cd-indices of all hypersimplices, extending a result of Stanley for the cd-indices of simplices. This research was carried out as part of the 2018 Combinatorics REU at the School of Mathematics at University of Minnesota, Twin Cities. We are grateful for the support of NSF RTG grant DMS-1745638.

168. The Combinatorics of Splitting and Splittable Families

Hao-Tong Yan Swarthmore College
Bryce Frederickson Utah State University
Samuel Mathers Princeton University
Advisor(s): Samuel Coskey, Boise State University

A set A is said to *split* a finite set B if exactly half the elements of B (up to rounding) are contained in A. We study the dual notions: (1) a *splitting family*, a collection of sets such that any subset of $\{1, \ldots, k\}$ is split by a set in the family, and (2) a *splittable family*, a collection of sets such that there is a single set A that splits each set in the family.

We study the minimum size of a splitting family on $\{1, \ldots, k\}$, using a variety of computational and theoretical techniques to calculate new values and bounds. We also study the structure of splitting families of minimum size, and as a result we find the exact value of the minimum size of special types of splitting families.

Next, we investigate splittable families that are in some sense on the edge of unsplittability. First, we study splittable families that have the fewest number of splitters, giving a complete characterization for the two set case and computational results for the three set case. Second, we define the *splitting game*, and study splitting under adversarial conditions. This work was done at the Mathematics REU at Boise State University.

169. Packing Patterns in Words

Eric Redmon Lewis University
Julia Krull Millikin University
Andrew Reimer-Berg Eastern Mennonite University
Advisor(s): Lara Pudwell, Valparaiso University

A word is an ordered list of numbers. Specifically, a permutation is a word without repeated letters, denoted π. A pattern is a word we look for within other words, denoted with ρ. We use π^r to represent the reverse of π. In general,

permutations are studied in terms of pattern avoidance, that is, which words avoid which patterns. Researchers have discovered several orderly ways to count pattern-avoiding words of the form $\pi\pi$ and $\pi\pi^r$.
Instead of avoiding patterns, we study pattern packing; that is, we identify words with as many copies of a pattern as possible. This idea was first studied by Burstein, Hästö, and Mansour, whose focus was packing patterns into general words, whereas ours is packing in words of the form $\pi\pi$ and $\pi\pi^r$. In particular, given a pattern ρ, we consider how many times we can pack ρ into words of these forms, what the ρ-optimal words look like, and how many ρ-optimal words exist for a given length of π. This work was completed as part of the Valparaiso Experience in Research for Undergraduate Mathematicians.

170. Stirling Numbers for Sunflower Graphs

Jose Garcia Grand Valley State University
Page Wilson Grand Valley State University
Matt Phad Grand Valley State University
Jessica Longo Grand Valley State University
Advisor(s): Lauren Keough, Grand Valley State University

Stirling numbers of the second kind are the number of ways to take n distinct elements from a set and put them into k distinct subsets, so that no subset is empty and the subsets are pairwise disjoint. To get the graphical Stirling number for a graph G, we add the restriction that any two vertices that are adjacent in G cannot be in the same subset. The traditional Stirling numbers of the second kind are the graphical Stirling number where the graph is empty. Using the relationship between the number of colorings of a graph and the graphical Stirling numbers, we explore the graphical Stirling number for sunflower graphs, which are generalization of stars. Sunflower graphs are denoted as $F^d_{p,\ell}$, where p is the number of 'petals' on the graph, ℓ is the length of each petal, and any two vertices on the same petal that are at most distance d are adjacent.

171. Wide and Latin Partitions

Carol Yaracz King's College
Advisor(s): Janine Janoski, King's College

A partition is wide if its subpartitions are in dominance order. A partition λ with parts $\lambda_1, \lambda_2, \ldots \lambda_k$ is Latin if the ith part of the Young Diagram can be numbered with a permutation of the integers one to the size of λ_i and each column numbered with distinct entries. In 2002, Chow et al. proved that Latin integer partitions are wide. We will discuss progress on proving that any wide integer partition is Latin. In particular, we will show a subcase of integer partitions for which this holds.

172. Point and Oval Incidences in Finite Projective Planes

Yunus Syed University of Illinois at Chicago
Advisor(s): Steven Senger, Missouri State University

We define an oval in a finite projective plane of order n, $FP(n)$, to be a collection of $n+1$ points such that no three points are collinear. Let O denote the set of all ovals in $FP(n)$, $P \subset FP(n)$ be a pointset and $I(P,O)$ denote the set of all incidences between ovals in O and points in P. Our primary objective is to provide bounds for $|I(P,O)|$. We will first determine a bound on $O(k)$, the number of ovals in O that contain a fixed set of k points, for any $k \leq n+1$. Using elementary arguments of similar style to those in Rudnev and Helfgott's paper on point-line incidences in projective planes, we then show a direct relationship between $|O(1)|$ and $|I(P,O)|$. Next, we determine bounds for $|O|$ through combinatorial methods so that. Lastly, the relationship between $|O(1)|$ and $|I(P,O)|$ allows us to relate $|O|$ to $|O(1)|$ and provide us with bounds for $|I(P,O)|$. This work was done as part of the 2018 Missouri State University REU.

173. Determinant formulas for counting linear extensions of tree posets

Stefan Grosser University of Massachusetts at Amherst
Advisor(s): Alejandro Morales, University of Massachusetts at Amherst

Various determinantal identities exist to count linear extensions of partially ordered sets. One set of examples are the Jacobi-Trudi identities. We ask the question: which posets have determinant representations for counting their linear extensions? We introduce a class of tree posets which all have determinant formulas. We then provide a q-analogue

and generalize this result to show there is a determinant formula for a class of posets which can be folded into a d-Complete poset.

174. The minimum permanent of doubly-stochastic matrices with restricted positions

Wilson Wang University of Massachusetts Amherst
Advisor(s): Alejandro Morales, University of Massachusetts Amherst

The Van der Waerden conjecture, stated in 1926 and proved in the 1980s independently by Gyires, Egorychev and Falikman, states that the minimum permanent of an n by n doubly stochastic matrix is $\frac{n!}{n^n}$ achieved only in the all $\frac{1}{n}$ matrix. We study the minimum permanent of doubly-stochastic matrices with restricted positions. We focused on two cases: tridiagonal doubly stochastic matrices and the support of the matrices in the Chan-Robbins-Yuen (CRY) polytope. Morales conjectured that the minimum permanent of such matrices is $\frac{1}{2^{n-1}}$. We settle this conjecture for the tridiagonal case and show it is only achieved in one such matrix and we make progress towards the Chan-Robbins-Yuen case. This is joint work with A. Morales in the REU program in University of Massachusetts Amherst.

175. Fractional Separation Dimension of Perfect Binary Trees

Shouzhuo Sun College of William and Mary
Advisor(s): Anke van Zuylen, College of William and Mary

The fractional separation dimension of a graph G, $\pi_f(G)$, defined by Loeb and West, is the minimum of a/b such that there exist a linear orderings (repetition allowed) which separate each pair of disjoint edges at least b times. We use integer linear programming and column generation to exactly determine $\pi_f(G)$ for perfect binary trees G. For perfect binary trees with height less than 4, we find one linear ordering that can separate all pairs of disjoint edges and thus $\pi_f(G) = 1$. For a height-4 perfect binary tree we show $\pi_f(G) = 6/5$ and for height-5 we have $5/4$. For perfect binary trees of arbitrary height k, there exist linear orderings that show $\pi_f(G) \leq 4/3$. To derive lower bounds on $\pi_f(G)$, we use the dual of the linear program, which allows us to conclude lower bounds by suggesting a non-negative weight function y for each pair of disjoint edges such that any ordering can separate at most a certain fraction of the total weight. Therefore, the inverse of the fraction will be the fractional separation dimension of G.

176. 3-Zebra Trees

Shakuan Frankson Howard University
Advisor(s): Dennis Davenport, Howard University

An ordered tree, also known as a plane tree or a planar tree, is defined recursively as having a root and an ordered set of subtrees. A 3-zebra tree is an ordered tree where all edges connected to the root (call this height 1) are tricolored as are all edges at odd height. The edges at even height are all black as usual. The companion concept is that of little 3-zebra trees where the tricolored edges are at even heights. We generalized our findings with k-zebra and little k-zebra trees, where k represents the arbitrary number of options to color an edge. We will show that the number of 3-zebra trees with n edges is the number of Schroder paths with bicolored level steps. Our future goal is to find a bijection for general k-zebra trees.

177. Row-Cyclic Latin Squares and Orthogonality

James Waldeck Marshall University
Advisor(s): Carl Mummert, Marshall University

We study the number of orthogonal mates of row-cyclic latin squares, and the graph of the mate relationship between row-cyclic squares of a fixed size. A latin square of size n is an $n \times n$ square where each number from 1 to n appears exactly once in each row and in each column. A latin square of size n is **row-cyclic** if the first column is an arbitrary permutation of $1, \ldots,$ and the values in each row increase cyclically, modulo n. Two squares are **orthogonal mates** if, when the squares are superimposed, every possible ordered pair of entries appears in the superimposed square. While the largest set of mutually orthogonal latin squares of size n can have no more than $n-1$ squares, a row-cyclic square of size n may have a much larger number of mates. All row-cyclic squares of the same size have the same number of row-cyclic mates. This means that the graph whose nodes are row-cyclic squares of a given size, and whose edges correspond to matehood, is vertex regular. We study additional properties of this mate graph, and investigate whether row-cyclic latin squares have potential applications to secret-sharing schemes.

178. Unavoidable colorful patterns

Alp Müyesser Carnegie Mellon University
Advisor(s): Wesley Pegden, Carnegie Mellon University

Let χ be a coloring of the edges of a complete graph on n vertices into r colors. We call χ ε-balanced if all color classes have ε fraction of the edges. Fix some graph H, together with an r-coloring of its edges. Consider the smallest natural number $R_\varepsilon^r(H)$ such that for all $n \geq R_\varepsilon^r(H)$, all ε-balanced χ of K_n contain a subgraph isomorphic to H in its coloring. Bollobás conjectured a simple characterization of H for which $R_\varepsilon^2(H)$ is finite, which was proved by Cutler and Montágh. Later, Fox and Sudakov improved this result by giving asymptotically tight bounds for the $r = 2$ case. Here, we obtain a characterization for arbitrary r, provide asymptotically tight bounds, as well as generalizations to infinite graphs.

179. On the Domination Number of Permutation Graphs and an Application to Strong Fixed Points

Peter Gardner Western Carolina University
Daniel McGinnis New College of Florida
Advisor(s): James Hammer, Cedar Crest College

A *permutation graph* G_π is a simple graph with vertices corresponding to the elements of π and an edge between i and j when i and j are inverted in π. A set of vertices D is said to $dominate$ a graph G when every vertex in G is either an element of D, or adjacent to an element of D. The *domination number* $\gamma(G)$ is defined as the cardinality of a minimum dominating set of G. In this article, we count the number of connected permutation graphs on n vertices with domination number 1 and domination number $\frac{n}{2}$. We further show that for a natural number $k \leq \frac{n}{2}$, there exists a connected permutation graph on n vertices with domination number k. We find a closed expression for the number of permutation graphs dominated by a set with two elements, and we find a closed expression for the number of permutation graphs efficiently dominated by any set of vertices. We conclude by providing an application of these results to strong fixed points, proving some conjectures posed on the OEIS. This work was completed at the 2018 REU program at Muhlenberg College.

180. Anti-van der Waerden results for $x_1 + x_2 = kx_3$ in $\mathbb{Z}_n$

Erin Bevilacqua Penn State
Samuel King University of Rochester
Suzannah Tebon Beloit College
Advisor(s): Michael Young, Iowa State University

In this work, we investigate the fewest number of colors needed to guarantee a rainbow triple of the equation $x_1 + x_2 = kx_3$ for cyclic groups $\mathbb{Z}_n$. This value is called the Rainbow number and is denoted by $rb(\mathbb{Z}_n, k)$ for positive integer values of n and k. First we consider the Schur equation ($k = 1$) and find that $rb(\mathbb{Z}_p, 1) = 4$ for all primes greater than 3 and that $rb(\mathbb{Z}_n, 1)$ can be calculated exactly from the prime factorization of n. For a general k we find the exact value of $rb(\mathbb{Z}_p, k)$, for every prime p and positive integer k. We also find that when k is prime, $rb(\mathbb{Z}_n, k)$ can be calculated exactly from the prime factorization of n.

181. The Collapsing Sum: Gaussian Blur on Arbitrary Matrices

Travis Dillon Lawrence University
Advisor(s): Samuel Gutekunst, Cornell University

Matrices are ubiquitous in mathematics. In addition to their use in describing linear transformations, they also lead to important results in algebra and combinatorics. We define a new function on matrices called the *collapsing sum*, motivated by the analysis of an efficient algorithm for applying the Gaussian blur technique from image filtering. We prove several structural results about this operation, including our first main theorem: the collapsing sum completely characterizes the winning positions of a one-player game. These results allow us to return to the Gaussian blur algorithm from a more mature perspective, and our second main theorem shows that applying the collapsing sum to a matrix is equivalent to applying the Gaussian blur. Finally, our third main theorem shows that this operation can be used to define arrays of integers which carry the structure of Riordan arrays, matrices that were recently introduced to unify topics in enumeration.

182. Representation Theory of the Rook and Symplectic Rook Monoid and their Hecke Algebras

Alexander Vetter Villanova University
Advisor(s): Benjamin Brubaker, University of Minnesota (Twin Cities)

The representation theory of finite groups is well studied; however, the representation theory of finite monoids is not. Inside every monoid is a group of units. We use the representation theory of the group of units in order to understand the representations of monoids. The irreducible representations of the symplectic rook monoid are indexed by partitions of at most n and by pairs of partitions whose sum is exactly n. We use combinatorial techniques to examine its character table and to develop branching rules for decomposing its irreducible representations as representations of the group of units. Using results from Solomon about the structure of the character table of the rook monoid, we determine a new way of producing the character table for the Iwahori-Hecke algebra of the rook monoid. In the spirit of Solomon and utilizing techniques from Geck and Pfeiffer, we provide a description of the character table of the symplectic rook monoid. We then extend this to the Iwahori-Hecke algebra of the symplectic rook monoid. This work was part of the University of Minnesota (Twin Cities) REU in Algebraic Combinatorics in 2018.

183. Multicolor Ramsey Numbers for Small Hypergraphs

Emily Zhu Carnegie Mellon University
Advisor(s): Tom Bohman, Carnegie Mellon University

In Ramsey Theory, a Ramsey number refers to the smallest size of a combinatorial object required to guarantee the existence of a certain substructure. For a multicolor Ramsey number, we consider k-coloring our object, often the edges of a complete graph, and seek a copy of our substructure in some color. The most natural multicolor Ramsey number for graphs is that for the triangle, where current bounds are still fairly loose–the lower bound is exponential while the upper bound is factorial. We study related problems for hypergraphs, where an r-uniform hypergraph is a system of groups of points where every group has exactly r points. Although asymptotically tight bounds are known for some multicolor Ramsey numbers of small 3-uniform hypergraphs, these bounds are not always exact. We consider this problem for a few more 3-uniform hypergraphs with 3 edges and provide new bounds for the multicolor Ramsey numbers as well as various stability results for the extremal families of some of these hypergraphs, including the giraffe and messy path.

184. The Pattern Doesnt Fall Far from the Tree: Exploring Patterns of the Calkin-Wilf 3-ary Tree

Michaela Deady Bridgewater State University
Advisor(s): Shannon Lockard, Bridgewater State University

The Calkin-Wilf tree was introduced in the paper "Recounting the Rationals" by Neil Calkin and Herbert Wilf. In it, they describe a binary tree in which each vertex is labelled by a rational number and then prove that these fractions enumerate the set of rational numbers. In addition to this enumeration, the tree has many interesting patterns and properties that have been explored by mathematicians. Recently, the tree was generalized to an m-ary tree where some of these patterns and properties no longer stay true. However, other new, interesting properties can be found among the fractions in the m-ary tree. In this presentation, we discuss the Calkin-Wilf 3-ary tree. We will show where the Calkin-Wilf binary tree fractions appear in the Calkin-Wilf 3-ary tree, determine how many times a certain fraction appears within any given level of the 3-ary tree, predict and describe the behavior of nonreduced fractions within the tree, and give a conjecture for a general description of any given level of the tree.

185. Deformations of the Weyl Character Formula for $SO(2n+1)$ via Ice Models

Alexandra Embry Indiana University
Sylvia Frank Amherst College
Advisor(s): Ben Brubaker, University of Minnesota

We study problems in combinatorial representation theory related to the Weyl character formula for complex Lie groups. In particular, we study highest weight characters of $SO(2n+1, \mathbb{C})$ using statistical mechanics. We explore properties of tetravalent graphs known as ice models, or six-vertex models, using the Yang-Baxter equation and similar relations. In particular, we show these ice models are in bijection with a variant of Gelfand-Tsetlin patterns, which are

in turn in bijection with certain shifted Young Tableaux. We then define a partition function over the admissible states of an ice model from a highest weight $\lambda = (\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \geq 1)$. These partition functions are shown to correspond to a deformation of the Weyl character formula for Cartan type B. We then show our partition function is a symmetric function by choosing Boltzmann weights that satisfy the Yang-Baxter equation. This work was completed at an NSF Funded Research Experience for Undergraduates at the University of Minnesota-Twin Cities.

186. Cyclic Sieving for Cyclic Codes

Shruthi Sridhar Princeton University
Advisor(s): Victor Reiner, University of Minnesota - Twin Cities

The Cyclic Sieving Phenomenon has been observed in many cases where a cyclic group acts on a finite set. In particular, it gives a generating function that counts the number of fixed points of the action. A triple $(X, X(t), C)$, consisting of a finite set X, a cyclic group $C = \{1, c, c^2, \ldots, c^{n-1}\}$ acting on X, and a polynomial $X(t)$ in $\mathbb{Z}[t]$, is said to exhibit the Cyclic Sieving Phenomenon if for every c^d in C, the number of x in X having $c^d(x) = x$ is given by the substitution $[X(t)]_{t=\zeta^d}$ where ζ is a primitive n^{th} root-of-unity. A cyclic code X is a subset of $\mathbb{F}_q^n$, such that if $(w_1, w_2, \ldots, w_n) \in X$, then $(w_n, w_1, \ldots, w_{n-1}) \in X$. In this project, the set X will be a specific class of cyclic codes called Dual Hamming Codes. Our cyclic group of n elements acts by cyclic permutation (shifting each entry to the right and the last one to the front). We show that, for Dual Hamming codes in certain fields $\mathbb{F}_q$, two important Mahonian polynomials are cyclic sieving polynomials.

187. Maximizing the number of vertices of the d-cube that can be covered by a ball of given radius

Oliver Meldrum Oberlin College
Advisor(s): Dezso Miklos, Renyi Intstitute

We consider the problem of finding the maximum number of vertices of a unit $d-$dimensional hypercube that can be covered by a hypersphere of radius r. We give solutions for $(d \leq 6)$ and $\left(r^2 < \frac{45}{44}\right)$ and provide some bounds on the solution in general. Finally, we disprove many natural conjectures, showing that this problem, despite it's elementary statement, appears to have a surprisingly complicated solution.

188. Special Configurations in Anchored Rectangle Packings

Vincent Bian Poolesville High School
Advisor(s): Tanya Khovanova, Massachusetts Institute of Technology

Given a finite set S in $[0, 1]^2$ including the origin, an anchored rectangle packing is a set of non-overlapping rectangles in the unit square where each rectangle has a point of S as its left-bottom corner and contains no point of S in its interior. Allen Freedman conjectured in the 1960's that one can always find an anchored rectangle packing with total area at least $1/2$. We verify the conjecture for point configurations whose relative positions belong to certain classes of permutations, as well as any configuration with 9 or fewer points.

189. Completely Separable and Multiple Choice Voters

Trung Nguyen Macalester College
Advisor(s): Andrew Beveridge, Macalester College

Separability is a desired property in voter preferences in a referendum on multiple issues. A voters preference for a subset of issues may or may not depend on the outcomes of others, which can cause undesirable final result when votes are tallied. We study this problem using a formal mathematical framework by looking at the space of total orderings on the power set of the issues set. In this framework, preferences are represented by vectors in a high dimensional space and constructed with a special basis, called the voter basis. Specifically, we explore the structure of the family of completely separable preferences (CSP) and extend our framework and the basis to the general case where each issue has non-binary possible options.

190. Recovering Conductances of Resistor Networks in a Punctured Disk

Yulia Alexandr Wesleyan University
Brian Burks University of California, Berkeley
Patricia Commins Carleton College
Advisor(s): Sunita Chepuri, University of Minnesota, Twin Cities

It has been proven by Curtis, Ingerman, and Morrow that for circular planar resistor networks, there exists a necessary and sufficient condition for recovering the conductance of each edge in the network uniquely from the response matrix. We generalize their results to certain types of resistor networks on a punctured disk. First, we define certain circular planar graphs that are electrically equivalent to standard graphs. We then turn them into networks on a punctured disk by adding a boundary vertex in the middle and prove such networks are recoverable. We then generalize this result to a much broader family of networks, thus obtaining a sufficient condition for recoverability. A necessary condition for recoverability is also introduced. We also prove several results about medial graphs of resistor networks on a punctured disk, define the notion of z-sequences for such graphs, and introduce new local moves. This research was performed at the University of Minnesota, Twin Cities 2018 Combinatorics REU.

191. Unique rectification in d-complete posets: towards the K-theory of Kac-Moody flag varieties

Michael Zlatin Rutgers University - New Brunswick
Advisor(s): Oliver Pechenik, University of Michigan

The jeu-de-taquin-based Littlewood-Richardson rule of H. Thomas and A. Yong (2009) for minuscule varieties has been extended in two orthogonal directions, either enriching the cohomology theory or else expanding the family of varieties considered. In one direction, A. Buch and M. Samuel (2016) developed a combinatorial theory of ‘unique rectification targets’ in minuscule posets to extend the Thomas-Yong rule from ordinary cohomology to K-theory. Separately, P.-E. Chaput and N. Perrin (2012) used the combinatorics of R. Proctor’s ‘d-complete posets’ to extend the Thomas-Yong rule from minuscule varieties to a broader class of Kac-Moody structure constants. We address the unification of these theories. Our main result is the existence of unique rectification targets in a large class of d-complete posets. From this result, we obtain conjectural positive combinatorial formulas for certain K-theoretic Schubert structure constants in the Kac-Moody setting. Work done with Rahul Ilango, Oliver Pechenik and Michael Zlatin during the summer at the DIMACS REU. Paper to appear in the *Electronic Journal of Combinatorics*.

192. On the Okounkov-Olshanski formula for the number of tableaux of skew shapes

Daniel Zhu Montgomery Blair High School
Advisor(s): Alejandro Morales, UMass Amherst

Standard Young tableaux of ordinary and skew shapes are fundamental objects in algebraic combinatorics and have applications to a variety of other fields. The hook-length formula of Frame, Robinson, and Thrall, proven in 1954, gives a product formula for the number of standard tableaux of ordinary shapes. However, there is no general product formula for skew shapes. In 1996, Okounkov and Olshanski found a nonnegative formula for the number of standard Young tableaux of a skew shape. We prove properties of this formula, including a determinantal formula for the number of nonzero terms and a q-analogue extending the Okounkov-Olshanski formula to reverse plane partitions. The latter result complements work by Chen and Stanley for semistandard tableaux of skew shape.

193. Identities Relating the Mullineux Involution with Generalized Regularization

Allen Wang Acton Boxborough Regional High School
Advisor(s): Guangyi Yue, Massachusetts Institute of Technology

From both a representation theoretic and combinatorial perspective, the Mullineux involution M_b can be considered as a possible b-analogue of transposition T. Walker, Bessenrodt, Olsson, and Xu used the regularization operator reg_b to prove an equivalent definition of M_b. After a second parameter was added by Yue and Dimakis to define $\mathrm{reg}_{a,b}$, we conjecture a sufficient and necessary condition that further generalizes previous work regarding M_b and reg_b. The proof of sufficiency is purely combinatorical. For necessity however, we evoke the notion of the Fock space of the quantum enveloping algebra $\mathcal{U}_q(\widehat{\mathfrak{sl}_b})$ of type $A_{b-1}^{(1)}$. We modify an algorithm computing the canonical basis of the basic representation of $\mathcal{U}$. When this basis is expressed in terms of the standard basis, the coefficients or the

q-decomposition numbers allow for an algebraic proof of necessity. At the project's completion we will not only have arrived at a simplification of the Mullineux operator, but also have found a more complete description of the q-decomposition numbers.

194. Investigating First Returns of Random Walks: The Effect of Multicolored Vectors

Myka Terry Morgan State University
Advisor(s): Dr. Leon Woodson, Morgan State University

By definition, a *first* return is the immediate moment that a path, using vectors in the Cartesian plane, touches the x-axis after leaving it previously from a given point; the initial point is often the origin. In this case, using certain diagonal and horizontal vectors while restricting the movements to the first quadrant will cause almost every first return to end at the point $(2n, 0)$, where $2n$ counts the equal number of up and down steps in a path. The exception will be explained further in the sections below. Using the first returns of Catalan, Schröder, and Motzkin numbers, which resulted from the lattice paths formed using a combination of diagonal and/or horizontal vectors, we then investigated the effect that coloring these vectors will have on each of their respective generating functions.

195. Tensor flattening approaches to estimate lower bound of small matrix multiplication tensor's border ranks

Yu Ma UC Berkeley
Advisor(s): Olga Holtz, UC Berkeley

Matrix multiplication efficiency lies at the heart of every computer algorithm. Strassen's algorithm initiated a new field of complexity theory looking for improvements on this problem. One popular approach in recent years is to evaluate the border ranks of tensor representations of matrix multiplications, written as $M_{\langle n,m,l\rangle}$. Unfortunately, even small sized problems such as determining the tensor rank of $M_{\langle 3,3,3\rangle}$ has not yet been solved. This project presents a detailed overview and applications of a newly emerged method, tensor flattening, that has shown to successfully assist evaluating lower bound of the border rank of $M_{\langle n,n,n\rangle}$. Furthermore, we specifically looked into the merits and limitations of Young flattening, proposed by Landsberg in 2013, with preliminary computational and theoretical investigation of its applications on small scaled rectangular matrix multiplication tensors. We end with a discussion of the implications of these results as well as the potential for generalization to other complexity problems.

196. Applying Q-Learning to Algorithmic Bitcoin Trading

Katherine Thai Rutgers University
Advisor(s): Queenie Lee, Hong Kong University of Science and Technology

Q-learning is a machine learning method used to teach an autonomous agent how to maximize its reward through its choice of actions in a given environment. We apply this technique to algorithmically trade between Bitcoin, a decentralized cryptocurrency, and US dollars. The Bitcoin market is very volatile and also very young, so many successful trading strategies have yet to be exploited. We show that a Q-learning strategy is able to out-perform the benchmark strategy of *buy-and-hold*, where one buys a maximal amount of Bitcoin and then makes no future trades.
We trained a *deep Q-learning* model on a combination of historical price data and a set of features we designed by analyzing the sentiment of Bitcoin-related news articles and correlating them with bearish and bullish markets. While the buy-and-hold strategy has a mean profit rate of -27.7%, our deep Q-learning model achieves a mean profit rate of 15.0% respectively.

197. Quantifying Conductivity Uncertainty in Transcranial Electrical Stimulation Simulations

Madison Guitard Roger Williams University
Advisor(s): Edward Dougherty, Roger Williams University

Transcranial electrical stimulation (TES) is a noninvasive treatment that has been proven to be a valuable therapy in treating neurological disorders. Mathematical modeling of this therapy has helped enhance treatment efficiency using simulations to predict electrical energy distributions within a head cavity. A fundamental drawback of these simulations is that precise, patient-specific cranial tissue conductivity values are unknown for particular patients, and can vary greatly between patients. To address this issue, we have implemented a stochastic mathematical model of

TES. Our governing system includes the Laplace Equation and boundary conditions that emulate TES electrodes. The finite element method is used to solve this system, and Monte Carlo experiments over tissue conductivities are utilized to quantify result variability from these stochastic conductivities. Simulations on an idealized domain and a MRI-derived head domain suggest that variability in tissue conductivities greatly impact results. We present our current results that showcase the importance of integrating conductivity uncertainty within mathematical models of TES.

198. Credit Risk Valuation Based on Machine Learning

Xuan Xu Tsinghua University
Yifan Wu Beijing Normal University
Advisor(s): Xiaodi Wang, Western Connecticut State University

An ideal credit risk evaluation model will accurately reflect the actual financial status of enterprises, which can offer a reasonable reference for financial firms in decision-making. Traditional credit grading systems are usually using linear combinations of several indexes or Black-Scholes formula based on stochastic process, weighted by empirical constants, such as Z and Zeta Models or KMV model; there was no good explanation about why those indexes, distribution and constants were made. In this research, by nonparametric machine learning methods, we provide evaluation algorithms, which iterate successively until reach a stable state, thus explain their rationality. First, we do not assume any index to be superior, while using logistic regression, hybrid genetic algorithm, recurrent neural networks and support vector regression, we may find the best indexes and weights to provide a reliable approximation of the single index of current ratio, which has its own inevitable inaccuracy and is easy to make fraud; then use normal distribution function to convert the result into probability scale. We believe that our model will outperform current models.

199. Analyzing the Performance of Sublinear CoSaMP

Simon Miller Oakland University
Jaya Blanchard Bowdoin College
Advisor(s): Mark Iwen, Michigan State University

We analyze compressive sensing as a way to learn functions of many variables and rapidly reconstruct signals using limited function evaluations. In high-dimensional problems, standard algorithms approximate the signal inefficiently due to high runtime and storage requirements. We explore various strategies for improving a specific compressive sensing algorithm, sublinear CoSaMP, in hopes of reducing runtime, error, and/or the number of samples required for the algorithm to run successfully. We also consider the algorithms performance on almost-sparse signals. Our work is an extension of the recent paper of Choi, Iwen, and Krahmer, *Sparse Harmonic Functions*, which discusses the improvements on the Support Identification step of CoSaMP. We would like to thank Lyman Briggs College at Michigan State University, the NSA, and the NSF for supporting this research through the 2018 SURIEM REU at Michigan State University.

200. Tile Sets Generated by Hadamard Submatrices of Fourier Matrices

Troy Wiegand Butler University
Advisor(s): John Herr, Butler University

Hadamard matrices are complex matrices where all entries are unit modulus and the matrix times its adjoint results in the identity matrix times the size of the matrix. Fourier matrices are Hadamard matrices where the entries have a specific structure based on its placement in the matrix. My work looks at specific relations between a Fourier matrix and all of its Hadamard submatrices and the relationships between the Hadamard submatrices that have the same column set. I looked at the tiling sets of the remaining rows for submatrices with same columns.

201. Interactive Treatment Planning in Cancer Radiotherapy

Wanxin Chen Temple University
Advisor(s): Abraham Abebe, Temple University

Intensity-Modulated Radiation Therapy(IMRT) is an advanced technique used in Cancer Radiotherapy. When a required amount of radiation is given to cancerous tumors of patients, radiation fields are expanded, potentially resulting in collateral damages to surrounding healthy tissues. The dose-volume histogram (DVH) is used to evaluate the effectiveness of a treatment plan. Literature concludes the approach of employing two and three moments formulation of

random variable to approximate the desired DVHs, suggesting improvements could be made. The goal of our work was to improve predictions about dosage requirements for future radiation sessions based on the original real-world DVH curves. We provided gradient information in Matlabs constrained minimization function, added an additional phase onto the algorithm with two and three moments and further shrunk the moments bounds after getting Pareto-optimal plan from two-phase approach. This has significantly improved the accuracy and predictability on the DVH ,which in turns, minimized treatment risk. We also determined that three moments were optimal by comparing reference and computed DVH curves.

202. Matrix Sparsification for Finding Alternative Basis Matrix Multiplication Algorithms

Nathan Cheng University of California, Berkeley
Advisor(s): Olga Holtz, University of California, Berkeley

A 2017 paper by Karstadt and Schwartz demonstrated how the computational complexity of an existing fast matrix multiplication algorithm could be sped up by roughly a constant factor by carrying out the multiplication in a different basis. The search for the optimal basis in which to carry out an algorithm (giving the best constant factor improvement) turns out to be equivalent to the NP-hard matrix sparsification problem. We present a novel matrix sparsification algorithm which is fast enough to optimally sparsify many small matrices in a feasible amount of time. We also present the improvements in computational efficiency gained from the optimal bases found by applying our sparsification algorithm to various fast matrix multiplication algorithms.

203. Implementing Machine Learning to Improve Bertini 2.0

Riley Anderson University of Mary Washington
Advisor(s): James Collins, University of Mary Washington

The purpose of this research is to decrease the run time of Bertini, a program that approximates roots of polynomial systems. Bertini can be run more efficiently if it is known whether a polynomial is singular or non-singular. In this research, we focus on polynomials in one variable. We use a machine learning algorithm to classify polynomials into these two categories. To do so, we create and use a set of polynomials to train a neural network and create a model. Then, we create and use a test set to assess the accuracy of the model. Through a process of training, evaluating, and changing the hyper-parameters of the neural network, such as the network architecture and learning rate, the accuracy of the model is able to be increased.

204. Predicting Parameters for Bertini Using Neural Networks

Makenzie Clower University of Mary Washington
Advisor(s): James Collins, University of Mary Washington

The purpose of this research is to use machine learning to predict the fastest settings for a program called Bertini. Bertini is a computer program that approximates solutions to systems of polynomial equations. The settings that were focused on were the differential equation predictor methods for step size when tracking the homotopy to the solution of the system. A neural network was used on a training set of data to create a model and then a test set was run through to obtain a percent accuracy for this model. Increased accuracy for the model was obtained by changing hyperparameters of the neural network. Neural networks with training sets of 3,000 and 8,000 were used and results were found.

205. The Role of Tortuosity in Filtration Efficiency

Ivan Mitevski Columbia University
Ines Vujkovac New Jersey Institute of Technology
Matthew Illingworth New Jersey Institute of Technology
Advisor(s): Ian Griffiths, Oxford University

Membrane fouling during particle filtration occurs through a variety of mechanisms, including internal pore clogging, coverage of pore entrances, and deposition on the membrane surface. Each of these fouling mechanisms results in a decline in the observed flow rate over time, and the decrease in filtration efficiency can be characterized by the volumetric flux, Q, as a function of the total volume of fluid processed, V. With asymmetric multilayered filters, comprising a series of membranes with constant pore sizes stacked on top of one another, filtration can be tailored in a variety of novel ways. We develop a network model that allows for a random pore distribution within the filter, which

captures the behavior of a globally connected filter in 3D. The model allows us to understand the relationship between tortuosity and efficiency, and establish when maximum efficiency can be expected, with respect to flux, and throughput. The filter is characterized through particle size, adhesivity to the membrane, pore size, and pore distribution, which allows for sweeps in parameter space that can be conducted to determine an optimal filter configuration for a given filtration challenge.

206. Fast and Stable Multivariate Numerical Rootfinding

Suzanna Stephenson Brigham Young University
Erik Parkinson Brigham Young University
Natalie Larsen Brigham Young University
Tyler Moncur Brigham Young University
Hayden Ringer Brigham Young University
Advisor(s): Tyler Jarvis, Brigham Young University

We present a multivariate numerical rootfinding algorithm that finds all real zeros in a given compact region in $\mathbb{C}^n$ of a system of functions. Our method builds on the ideas of Nakatsukasa, Noferini, and Townsend of subdividing the original search interval and approximating the functions with Chebyshev polynomials. We then use a variant of the method of Telen and van Barel, finding the roots in each subdomain by computing eigenvectors of the Chebyshev form of certain Möller-Stetter matrices constructed with a well-chosen basis. We compare our algorithm, in terms of accuracy and speed, to other popular numerical rootfinding algorithms. In many instances, this algorithm outperforms all known competitors.

207. A HoTT Approach to Computational Effects

Phillip Wells College of Wooster
Advisor(s): Nathan Fox, College of Wooster

A computational effect is any mutation of real-world state that occurs as the result of a computation. We develop a model for describing computational effects within homotopy type theory, a variety of intensional dependent type theory that equates the identity type of objects with the type of isomorphisms between those objects. Such a model allows us to describe programs as total functions over values while preserving information about the effects those programs induce.

208. Graph Based Algorithms for Non-negative Matrix Factorization

Nitya Raju Carnegie Mellon University
Advisor(s): Jason Howell, Carnegie Mellon University

Non-negative matrix factorization is a commonly used tool in feature extraction and has various uses in machine learning. The problem is as follows: given non-negative matrix $V \in \mathbb{R}^{n \times m}$ and parameter $p \in \mathbb{R}$ such that $p << m, n$ factor V into two non-negative matrices $W \in \mathbb{R}^{n \times p}$ and $H \in \mathbb{R}^{p \times m}$ such that $WH = V$. Exact non-negative matrix factorization is known to be NP-Hard to compute, hence approximation algorithms that are poly-time are important. Current approximation algorithms use multiplicative update rules to adjust a random initial iterate to approximate WH to be within a given margin of error. We aim to use a bipartite graph representation of non-negative matrices to develop deterministic approximation algorithms for non-negative matrix factorization.

209. A Preliminary Study: The Use of Predictive Modeling and Text Mining to Classify Fast Food Service Failures

Nathan Ramsey East Tennessee State University
Advisor(s): Michele Lynn Joyner, East Tennessee

Given the variety of social media platforms, text data can provide a wealth of new information which companies can leverage to analyze customer satisfaction. The ultimate goal in this poster is to use predictive modeling with text mining techniques to aid fast food restaurants in identifying those service failures which require the most attention given text data (such as that from a social media platform). In this project, we explore standard text mining skills in Python to first analyze the sentiment of a text using labeled training data. We then examine how one might use this sentiment analysis with labeled data from fast food text reviews indicating a given type of service failure to help identify those service failures which might affect customer satisfaction the most.

210. Stochastic Single-Lane Vehicular Traffic Flow Model Analysis

Alexander Clark College of the Holy Cross
Advisor(s): Eric Ruggieri, College of the Holy Cross

Traffic is at the root of every persons day-to-day life. Understanding its behavior could be a key step in determining the effectiveness of different types of intersections and road segments. In this project, we simulated traffic flow in numerous scenarios, including, straight road segments, rotary and stop-light behavior, and random collision segments. Our model is based on the work of Kai Nagel and Michael Schreckenberg who presented a simplified model of traffic flow in the paper A cellular automaton model for freeway traffic. We implemented a stochastic model, using elements of randomness in deceleration behavior, collision occurrence, red-light frequency, and other variables. Real time, single-lane traffic data was used to check the accuracy of our model, and of conditional traffic behavior. Our model is shown to capture many important features of real-traffic patterns.

211. Parallel Deterministic Frames for Compressed Sensing

David Neill Asanza Grinnell College
Advisor(s): Jeff Blanchard, Grinnell College

Compressed sensing is a branch of signal processing focused on the efficient reconstruction of a signal from a relatively small number of measurements. Compressed sensing has applications in digital photography, MRI, astronomy, and DNA sequencing. While some non-deterministic frames were proven to produce high probability of recovering a sparse vector, Monajemi et al. showed similar recovery with certain families of deterministic matrices associated with efficient storage and fast matrix-vector multiplication algorithms. This project focuses on the implementation of eight deterministic matrix transformations in the CUDA C programming language to exploit the parallel processing power of Nvidia GPUs. We show that our implementation reproduces the recovery phase transitions expected from the literature. The GPU code achieves a 4x-100x speed-up over the serial CPU implementation. The reduced runtime enables solving problem sizes with signal lengths in the millions, which were previously prohibitively time-consuming. The project represents a significant contribution to the field of compressed sensing, enabling application and research at reduced runtime and larger problem sizes.

212. The Hausdorff Dimension of the Limit Set of a Pair of Pants

Ian Hill James Madison University
Francisco Castaeda Instituto Tecnolgico Autnomo de Mxico
Advisor(s): Moira Chas, Stony Brook University

Consider a surface with no genus and three boundary components, this is commonly referred to as a pair of pants. We know that the metrics naturally fitting on this surface are hyperbolic. Furthermore, the space of metrics on the pair of pants is three dimensional and correspond exactly to the lengths of the boundary components. From this, we can define a transformation group and consider its limit set. The Hausdorff dimension of this limit set is known to be of importance when analyzing the number of curves of a given hyperbolic length that live on the surface. We implement an algorithm designed by Curt McMullen to numerically approximate the dimension and show our result. This work was done in the Summer@ICERM 2018: Low Dimensional Topology and Geometry program.

213. Automatic Monte Carlo Methods for Bayesian Inference

Noah Grudowski Illinois Institute of Technology
Advisor(s): Fred Hickernell, Illinois Institute of Technology

Bayesian inference is an efficient way to estimate the parameters of a distribution. However, a problem arises when looking at Bayesian inference posterior mean estimators for coefficients of regression. These estimators involve a ratio of multidimensional integrals that cannot be calculated analytically. Therefore, the aim of this study is to use automatic quasi-Monte Carlo methods to both accurately and efficiently estimate these ratios. After performing the appropriate transformations, these estimators can be approximated using quasi-Monte Carlo cubature. In order to do this, we must sample from some suitable density . The three densities that we chose to sample from are the prior, the Gaussian approximation to the likelihood, and a product of the two. We found that we were able to successfully obtain estimates, but our choice of density directly affected the time that each estimate took. Specifically, choosing to sample from the prior consistently gave us the longest run time. Therefore, sampling from either the Gaussian approximation to the likelihood or a product of the prior and likelihood seems to be more efficient when approximating posterior mean estimates.

214. TActIC: Tanh Activations in Image Classification

Heyley Gatewood Stetson University
Samuel Hood Morehouse College
Jonathon Scott Macalester College
Advisor(s): David Uminsky, University of San Francisco

Our research is focused on applying deep learning to the classification of images that have been corrupted with Poisson noise, an electronic noise that occurs when the energy levels of the technology producing the image are low, revealing differing variations of pixel quality within the image itself. We implement multiple deep learning neural network architectures using PyTorch. For training and testing, we use the original Modified National Institute of Standards and Technology (MNIST) data set and the corrupted MNIST data sets. An important part of optimizing performance in a neural network is choosing an activation function; for, certain activation functions allow deep learning models to achieve higher or lower accuracy measurements. We propose two new trainable activation functions, which we call GTAct and TAct. Then, we compare the performance of GTAct and TAct to other activation functions, including ReLU and Swishthe activation function that has recently claimed to be the most successfulby testing each function each of our data sets. Our experiments show that GTAct and TAct are the best activation functions for image classification, outperforming RELU, Swish, and more.

215. An Investigation of Carmichael Number Sequences

Edie Johnson Riverstone International School
Abigail Chen Boise High School
Catherine Ji Capital High School
Andrew Every Boise High School
Mitchell Messerley Borah High Schoo
Advisor(s): Liljana Babinkostova, Boise State University

Accurately identifying large prime numbers is essential for implementing computationally hard problems to protect digital information. Pseudoprimes are composite numbers that pass a primality test. Carmichael numbers are pseudoprimes that pass Fermats primality test. Our results include an algorithm that successfully differentiates Carmichael numbers from prime numbers. Using the notion of quadratic residues we introduce and investigate several sequences of Carmichael numbers. A conjecture on conditions that imply which Carmichael numbers have at least four prime factors emerged from our work.

216. Counting Sheep: Why Sleep Apnea is a Real Concern to Individuals with Down Syndrome

Graham Brooks Simpson College
Mason Remington Simpson College
Levi Lefebure Simpson College
Advisor(s): Heidi Berger, Simpson College

Objective—Down syndrome Disintegrative Disorder (DSDD) is newly emerging in clinical literature. Most of the studies, however, only focus on the appearance of this disorder, and not the etiology. Our objective is to explore the relationship between sleep apnea and DSDD. Methods—A survey monkey survey was emailed out to patients at Massachusetts General Hospital's Down syndrome (DS) clinic and posted on Facebook. The survey gathered mostly qualitative data with yes/no, multiple choice, and open-ended questions. Results—Of the 191 respondents (27%), 115 were aged 10–35. These are the ages where DSDD is commonly diagnosed. For these two groups, we saw a statistically significant difference in the number of hospitalizations, whether or not they have regressed, and their mean age of regression. Conclusion—There is a relationship between sleep apnea and DSDD.

217. Tanh Activations in Image Classificaton

Samuel Hood Morehouse College
Heyley Gatewood Stetson University
Jonathon Scott Macalester College
Advisor(s): Mario Banuelos, Fresno State

In machine learning, feature selection is important in being able to distinguish variables from each other. A subset of machine learning, deep learning, incorporates nonlinearity through activation functions – these functions allow us to transform an input signal from a node to an output signal for the next layer. Our work focuses on an exploration of features and activation functions in neural network architectures. This lead us to study the hidden layers of a neural network on noisy imaging data. We are interested in the hidden layers because knowing how they function will help us understand how a computer learns. In particular, we use linearization of activation functions in our models and compare training and test errors as well as the computational costs of each method. We incorporate noisy representations of MNIST data in our models into the input layer by adding Poisson noise. Poisson noise was used because the distribution of photons captured by medical devices closely mimics a Poisson distribution. Mathematical Sciences Research Institute

218. Extension of Bass Diffusion Model

Nils Lehmann Rollins College
Advisor(s): Zeynep Teymuroglu, Rollins College

The Bass Diffusion Model has been used extensively in marketing to study the spread of innovations. The core assumption is that initial adoption is linearly dependent on the number of previous adopters. Additionally, the model assumes innovative and imitative behavior stay constant and do not change over time. Valente et al. (2015) hypothesize on the one hand that innovative behavior decreases linearly as more people adopt an innovation and on the other hand that imitation influence increases over time as aspects such as persuasion and word to mouth play a more dominant role. Our research aims to generalize the original Bass Model by including more realistic innovation and imitation processes.

219. Topological Structure of Reaction-Diffusion Systems

DeAndre Johnson Virginia State University
Advisor(s): Junping Shi, The College of William and Mary

Topological Structures of Reaction-Diffusion Systems consist of me exploring the interpretation through mathematical methods. Methods such as, the finite difference, and the use of rare mathematical branch known as algebraic topology. The result received through mathematical computation was transfered from solutions (concentration maps) to a time series plot that recorded the trends that would somewhat fluctuate as time went on.

220. Clairaut's Equation on Arbitrary Time Scales

Hunter Hartke UW-Eau Claire
Melany Puser UW-Eau Claire
Advisor(s): Christopher Ahrendt, UW-Eau Claire

For this project, we studied Clairaut's equation, defined as $y = ty^{\Delta} + f(y^{\Delta})$, on arbitrary time scales, more specifically on isolated time scales. We compared our results to the solutions found when solving the classic Clairaut equation in the real numbers. In the case of $f(x) = x^3$, we solved Clairaut's equation, and we found a general solution and a rather complicated singular solution. We identified regions in both the solution domain and phase portraits to help us analyze how the solution would behave given a particular initial condition. In particular, using the substitution $v = y^{\Delta}$, we explored various regions of behavior in the t vs. v plot. Then, we mapped the regions onto two different phase portraits so that we could visualize the behavior of the solutions. Finally, we graphed our regions on the t vs. y plot and compared the solution behavior to the classic case of the Clairaut equation.

221. Breather Soliton Interactions for the Quaternionic KdV Equation

John Cobb College of Charleston
Advisor(s): Alex Kasman, College of Charleston

The KdV equation is fundamental in the description of a wide array of physical phenomena. It remains the prototypical example of a completely integrable nonlinear partial differential equation because of its *n-soliton* solutions, which appear to be composed of n traveling waves that collide in particle-like fashions. Despite this fame, little has been said about KdV solitons in a noncommutative setting. In this project, Darboux transformations were used to produce quaternion-valued solutions to the non-commutative KdV equation. I will describe the nonlinear superposition principle governing the interactions of the breather soliton solutions with other solutions including rational and periodic solutions. Finally, I will examine the kinetics of the general 2-soliton interaction, including a formula for the phase shift which, unlike the commutative case, is not determined by the wave numbers of the constituent 1-solitons.

222. Waves: A Mobile App for Exploring Partial Differential Equations

Frederick Jacques Joubert Pepperdine University
David Nicholas Pepperdine University
Jereld Chng Pepperdine University
Advisor(s): Timothy Lucas, Pepperdine University

Waves is an app for iPhone and iPad with three activities currently in development at Pepperdine University due for release in Spring 2019. The *Fourier* activity plots Fourier series by designating either the coefficients or the function to be approximated. The activity contains a sliders to increase the number of terms and adjust the coefficients. The *Diffusion* activity focuses on the behavior of solutions to the one-dimensional heat equation. Users can specify the boundary and initial conditions to animate the solutions over time. Initial conditions can be specified using an equation or by drawing a function directly on the screen. A future activity will plot and animate solutions to the one-dimensional wave equation. All three activities come preloaded with interesting examples and allow users to export their work. The app was designed with simplicity as a focus to take full advantage of the touch interactions available on mobile devices. Traditional mathematical software often has a steep learning curve that requires users to understand a complicated syntax, but *Waves* allows users to engage the world of partial differential equations in an intuitive way.

223. Numerical Solutions to Nonlinear Boundary Value Problems Using Bernstein Polynomial Reproducing Kernel Method

Alec Wendland Carroll University
Advisor(s): Thomas St. George, Carroll University

In this project, we studied and advanced a reproducing kernel method for solving systems of nonlinear boundary value problems numerically. In doing so, a reproducing kernel function is constructed on a finite dimensional reproducing kernel Hilbert space spanned by a basis of Bernstein polynomials of degree m. Using only a finite sequence of nodal points in a given domain, our numerical solution $u_m(x)$ is then expressed as a finite sum of basis functions generated by the reproducing kernel function in the solution space of the system. The presented procedure is applied to example systems to demonstrate that our method is simpler and more accurate than previously researched methods.

224. Comparison of the Effects of Mixed Delay/Instantaneous Terms on the Frequency of Delay Oscillator

Kalsang Sherpa Trinity College
Advisor(s): Lauren Lazarus, Trinity College

This research investigates the dynamics of an oscillator modelled by a delay differential equation under periodic external forcing, comparing variations on the systems cubic term. Perturbation methods were applied to these systems giving rise to a slow flow system of ordinary differential equations rather than delayed. Through linear stability analysis of the slow flow system, we find Hopf and saddle node bifurcations indicating behaviors of quasiperiodic and periodic motion for different parameter values. The findings show that the number of delay terms in the cubic term impact the nature of the oscillator, primarily its natural frequencies, and how it responds to the forcing term.

225. Global Solution to a Non-linear Wave Equation of Liquid Crystal in the Constant Electric Field

Linjun Huang UC Davis
Advisor(s): Qingtian Zhang, UC Davis

We construct a global conservative weak solution to the Cauchy problem for the non-linear variational wave equation $v_{tt} - c(v)(c(v)v_x)_x + \frac{1}{2}(v + v^3) = 0$ where $c(\cdot)$ is any smooth function with uniformly positive bounded value. This wave equation is derived from a wave system modelling nematic liquid crystals in a constant electric field.

226. Stable Annulus Solution for the Diblock Copolymer Equation

Micheal Belete George Mason University
Advisor(s): Thomas Wanner, George Mason University

Diblock copolymers are designer materials exhibiting microphase separation. Motivated by experimental observations, we study the occurrence of annulus microstructures which are equilibrium solutions for the underlying partial differ-

ential equation. We numerically discuss their stability and parameter-dependence. This work was done as part of the George Mason University EXTREEMS program.

227. Dynamics of a Stage-Structured Population Model with Allee Effect and Asymmetric Dispersal

Peter Psathas College of William and Mary
Advisor(s): Leah Shaw, College of William and Mary

Population dynamics models have proved to be crucial in guiding restoration efforts for populations such as oysters. We introduce a simplified stage-structured model based on two distinct phases in the life cycle of an oyster, juvenile and adult, and study a two-patch system. New juveniles arise from dispersal, which can be asymmetric due to water currents, and adults display an Allee effect. We find that as the dispersal of juveniles between the two patches becomes more symmetric, both patches can persist at lower growth parameter values. However, as the growth parameter of the two patches approaches the minimum value necessary for persistence, the encroachment zone in which only the extinction equilibrium exists increases in size. Lastly, we modify our model to include a different growth parameter for each patch to account for thriving and failing patches, that is, with growth parameters above and below the minimum threshold value for persistence, respectively. Our research shows that it is possible for a thriving patch to rescue a failing patch by supplying it with juveniles.

228. The Talbot Effect for Dispersive Partial Differential Equations

Jack Bernard University of Illinois at Urbana Champaign
Bowen Song University of Illinois at Urbana-Champaign
Advisor(s): Burak Erdogan, University of Illinois at Urbana Champaign

The solution profile of dispersive equations posed on periodic domains depend heavily on the algebraic properties of time, a phenomenon called the Talbot Effect in the literature. In particular, the solution can be a continuous fractal curve at irrational times and a simple step function at rational times. In this project we set up numerical experiments to study this behavior for model equations such as the Schrodinger and the Airy equations. We are especially interested in quantifying this behavior by numerical evaluation of the box dimension of the solution curves.

229. Scalability of Integrating Factor in Solving Non-Homogenous Higher Order Ordinary Differential Equations

Huy "Simon" T. Vuong Pasadena City College
Advisor(s): Pete "Edward" Riley, Pasadena City College

The concept of Integrating Factor is originally developed to turn any First Order Ordinary Differential Equation into solvable forms. However, it is very interesting that by applying Integrating Factor in multiple stages, higher order and more complex Ordinary Differential Equation can be solved successfully in the similar manner and philosophy. Such approaches are generalized for the cases of Non-Homogenous Linear Ordinary Differential Equations and Eulers Differential Equations with high scalability and flexibilitysuch algorithm is independent of the right-hand-side function. The author believes that the result is a break-through in Mathematics Education at college level since it helps simplify and lessen the formula/knowledge required to successfully negotiate these two fairly common types of Ordinary Differential Equations.

230. Dynamical Models of Early Shoot Growth

Michael Zhang Wartburg College
Tomas Bryan University of Houston
Advisor(s): Sergiy Koshkin, University of Houston

In this paper, we study some simplified versions of the Bessonov–Volpert model of plant growth using numerical simulations and statistical analysis. The growth is driven by dynamics of interaction between inflow of the nutrients from the soil and the growth hormone, but we replace the partial differential equation for the transport of the nutrient with an ordinary differential equation. As in the original model, there are two possible regimes, linear growth and growth spurts interspersed by quiet periods. In both cases the growth eventually stops, but the final length depends on initial values in addition to the parameters of the model. The final length and the time it takes to reach it strongly

depend on all parameters, especially the diffusion coefficient and the rate of hormone absorption. We also discuss how different growth regimes affect the distribution of auxillary buds (proto-leaves) along the shoot. We would like to thank Dr. Sergiy Koshkin for helping us conduct this research and the University of Houston–Downtown, NSF Grant #1560401 for funding this research.

231. Partial Results in the Nivat Conjecture

Eben Blaisdell Bucknell University
Advisor(s): Van Cyr, Bucknell University

Consider colorings of the discrete line, i.e. the integers. We define the n-scale block complexity of such a coloring to be the number of ways to color n points in a row. Thus, the scale 5 block complexity of ...RBRBRBRBR... is 2 because the only ways to color 5 points in a row are RBRBR and BRBRB. The Morse-Hedlund theorem from the 1950s states that the n-scale block complexity is at most n if and only if the coloring repeats itself in n or fewer steps. The Nivat conjecture from the 1990s makes a similar claim in the discrete plane, $\mathbb{Z}^2$. A few partial results of the Nivat conjecture are known that were proven through dynamical means. In a recent paper, Kari and Szabados used algebraic geometry to force a very useful decomposition property onto certain two-dimensional colorings. By utilizing tools from algebra, discrete geometry, and dynamical systems, I improved this decomposition, essentially finitizing it. This improvement imposes dynamical properties, and thus bridges the algebraic geometry of the recent paper with the past dynamical partial results. This improvement additionally answers a question of expansiveness posed by Kari and Szabados themselves.

232. Decomposition of Nonlinear System Dynamics into Multiple Time Scales

Ryan Chakmak Claremont McKenna College
Colleen Chan Yale University
Gal Dimand University of Redlands
Aaron George University of Maryland
Advisor(s): Claudia Falcon, University of California, Los Angeles

The United States Air Force Research Laboratory at Edwards Air Force Base investigates how Hall thrusters are used to stabilize spacecraft orbits. The physics of these thrusters are determined by chaotic systems, where slight perturbations in initial conditions lead to unpredictable results. In the case of Hall thrusters, experimental data suggests there is an interference of either noise or signal. Since this data is determined by nonlinear dynamics, traditional methods such as the Fourier transform fail. We present an algorithm which takes two causally-related signals and separates them from their interference. This process is an extension of the "convergent cross mapping" (CCM) technique developed by Sugihara et al. in 2012. We extend CCM to reconstruct signals while adding implementations of ways to deterministically select optimal tuning parameters. We find that while our method fails to outperform traditional smoothing methods on noisy signals, it succeeds on separating a composite signal into its parts. This algorithm is then applied to analyze experimental Hall thruster data, from which we are able to recreate two distinct constituent signals.

233. Graph Replacement Systems for Julia Sets of Quadratic Polynomials

Yuan Liu Bard College
Advisor(s): James Belk, University of St. Andrews

Belk and Forrest construct a specific class of graph replacement systems that give sequences of graphs that converge to fractals. Given a complex polynomial p, we have an algorithm that gives a replacement system that leads to a graph sequence which we conjecture converges to the Julia set of p. We prove the conjecture for the quadratic polynomial $z^2 + c$ where c is a real number and the critical point is in a three cycle. We present some additional results and observations on replacement systems obtained from certain polynomials.

234. To Spin or not to Spin: Simulating Wheel of Fortune in Python

Billy Heidel Heidel Stevenson University
Advisor(s): Benjamin Wilson, Stevenson University

Using programs written in Python, we analyze the complexity of various texts using measures such as n-gram frequency and entropy. N-gram frequency is the probability of a string of n letters. These probabilities are used to calculate the entropy, or unpredictability, of the text. With these measures, we simulate phrase guessing games similar to

Wheel of Fortune or Hangman. The goal is to see how effectively the program can guess a phrase given that its "language" is based on various texts. These texts include the Bible, Book of Mormon, a collection of Wikipedia articles, a collection of words from a dictionary, Presidents' addresses, tweets from celebrities, a collection of literary works by various authors including William Shakespeare, and a collection of Wheel of Fortune puzzle solutions. We compare these corpora's ability to solve Wheel of Fortune puzzles by pitting them against each other, running this program in various tournaments to determine which of the corpora is the best at Wheel of Fortune and why.

235. Non-Rigid Rank-One Infinite Measures on the Circle

Hindy Drillick Stony Brook University
Alonso Espinosa-Dominguez Massachusetts Institute of Technology
Jennifer N. Jones-Baro Universidad de Guanajuato/CIMAT
James Leng University of California, Berkeley
Yelena Mandelshtam Stanford University
Advisor(s): Cesar Silva, Williams College

Rank-one transformations have played an important role as a source of examples and counterexamples in ergodic theory. In 1976, del Junco showed that irrational rotations are rank-one, but did not give an explicit cutting and stacking construction. For a class of irrational numbers, depending on their Diophantine properties, we construct explicit rank-one transformations that are totally ergodic and not weakly mixing. We classify when the measure is finite or infinite. In the finite case they are isomorphic to irrational rotations, and we obtain explicit cutting and stacking constructions for these transformations. In addition, we extend this construction to obtain rank-one non-rigid infinite invariant measures on irrational rotations. One consequence is getting the first examples of infinite measure rank-one transformations that are totally ergodic and not weakly mixing. We also obtain nonsingular measures not admitting an invariant measure for irrational rotations. This research was done at the 2018 SMALL REU at Williams College.

236. Affine Kaprekar Algorithm

Ryan Valentin Ithaca College
Advisor(s): David Brown, Ithaca College

In this project, we investigate the dynamical properties of affine versions of the well-known Kaprekar algorithm for four-digit integers, with significant focus on three-digit integers. In particular, we determined the cycle structures for the various algorithms in base-10 and other bases as well as incorporating the addition of a constant. Through understanding the patterns among the various bases, we are able to clearly define fixed points and cycles for odd and even bases. The goal of focusing on the three-digit integers is to later use these patterns and concepts as stepping stones for tackling four-digit integers, which hold new and exciting properties not seen in three-digit integers.

237. Matrices with Oscillating Orbits

Oana Prajitura SUNY Brockport
Advisor(s): Ruhan Zhao, SUNY Brockport

We study matrices A for which there are vectors x such that $\liminf ||A^n x|| < \limsup ||A^n x||$. This type of oscillating behavior is related to Li - Yorke chaos.

238. Piecewise Translations on a Symmetrically Partitioned Plane

Jaaziel Lopez de la Luz University of California, Irvine
Advisor(s): Anton Gorodetski, University of California, Irvine

Piecewise isometries are a relatively new topic in mathematics with several applications in electrical engineering and data filtering. A piecewise isometry could be primed by the following analogy: imagine splitting a piece of paper into several shapes and marking their position and assigning a set of translations or rotations to each position; we will move each shape according to its position on the plane. Repeating the process, we begin to form an image that has self-resemblance after several iterations. In this presentation, we provide a generic algorithm for a specific case of translations on a symmetric partitioned plane and prove instances of periodicity.

239. Analysis of Kermack-McKendrick Models

Darreon Phipps Morehouse College
Advisor(s): Alexandru Alin Pogan, Miami University

First, we consider the classical Kermack-McKendrick Model, that ignores the population growth and assumes that infected individuals cannot recover. We also consider a generalized Kermack-McKendrick model, which is modified to account for incremental and decremented rates of infected/susceptible individuals due to physical contact with infected individuals and epidemic prevention efforts. To prove our results we use dynamical systems methods, in particular phase-plane analysis. We graph the solutions using the Runge-Kutta method. This research was conducted at Miami University.

240. Hyperbolic Julia and Mandelbrot Sets

Aaron Shukert Colorado State University
Advisor(s): Patrick Shipman, Colorado State University

Julia sets and Mandelbrot sets are fractal subsets of the complex plane that capture properties of discrete dynamical systems given by quadratic polynomials. In this work, we investigate analogous sets for dynamics of hyperbolic numbers. Hyperbolic numbers have the form $a + tb$, where a and b are real, and the square of t is 1, but t is not equal to 1 or -1. We use similar algorithms to the complex number is order to compute Julia Sets and the Hyperbolic Mandelbrot set. In contrast to the typical complex case, however, we are able to analyze and find exact forms for the hyperbolic analogs of Julia Sets and Mandelbrot sets using a coordinate transformation to characteristic coordinates and the logistic map. This work was done as part of an REU program at Colorado State University.

241. Models on the unit square of the Chacón, Pascal, and other cutting and stacking transformations

Jennifer N. Jones-Baro Universidad de Guanajuato/CIMAT
Hindy Drillick Stony Brook University
Alonso Espinosa-Dominguez Massachusetts Institute of Technology
James Leng University of California, Berkeley
Yelena Mandelshtam Stanford University
Advisor(s): Csar E. Silva, Williams College

The Chacón transformation on the unit interval is an important example in ergodic theory that has been a source of many examples and counterexamples; in particular, it is a measure-preserving transformation that is weakly mixing but not mixing. Likewise, the Pascal transformation is another important transformation, but many questions about its dynamical properties still remain open. We construct transformations on the unit square that are piecewise translations on rectangles and are isomorphic to the Chacón and the Pascal transformations, and then generalize the constructions to any n-dimensional square. This construction can be used for other transformations such as rank-one transformations. Having a construction in two dimensions allows us to visualize the dynamics of the transformations, and we have constructed some animations through which some dynamical properties are observed. This allows us to make numerical estimations and new conjectures. This project was done at the 2018 SMALL REU at Williams College.

242. Mathematical model of coupled-patch population system

Xinyao Wang College of William Mary
Advisor(s): Sarah Day, College of William & Mary

Coupled patch model could be used to analyze the dynamics of populations. In my research, I focus on the two-dimensional spatial model with two parameters. Since I choose to examine the dynamics of population, my two parameters are growth rate and dispersal rate. Growth rate refers to the dynamical change of population in the area itself, while dispersal rate refers to the dynamics of interactions between neighboring areas.

243. Mathematical Measures of Fairness in Legislative Districting

Bryson Kagy Georgia Institute of Technology
Brady Gales Wake Forest University
Advisor(s): David Offner, Westminster College

Gerrymandering is the term used to describe the drawing of legislative districts to favor one party over another. Recently, many mathematicians have tried to develop mathematical tools to decide if legislative districts are gerrymandered, and define fair methods of districting. For example, Landau, Reid, and Yershov propose a protocol for districting based on a two-player fair-division process, where each player is entitled to draw the districts for a portion of the state. We call this the LRY protocol. Landau and Su propose a measure of the fairness of a districting called the geometric target. We prove that the number of districts a party can win under the LRY protocol can be at most two fewer than their geometric target, assuming no geometric constraints on the districts. This is the first quantitative bound of this type and we provide examples to prove this bound is tight. We also show that the protocol on a state drawn with geometric constraints can produce an unbounded difference from the geometric target. This research was preformed at Carnegie Mellon University as part of SUAMI.

244. The One-Seventh Ellipse Problem

Shida Jing Grinnell College
Sanah Suri Grinnell College
Advisor(s): Marc Chamberland, Grinnell College

A curious mathematical phenomenon is called the *One-Seveth Ellipse*. Take the digits from the decimal expansion of $1/7$, namely 142857, and form six points in the plane: $(1,4)$, $(4,2)$, $(2,8)$, $(8,5)$, $(5,7)$ and $(7,1)$. The surprising fact is that these six points lie on an ellipse. Moreover, using consecutive digits from the decimal expansion, consider the points $(14,42)$, $(42,28)$, $(28,85)$, $(85,57)$, $(57,71)$ and $(71,14)$. These six points also lie on an ellipse. In this project, we show how to generalize this phenomenon extensively.

245. Investigating shortest paths in generalizations of the Cantor Set

Elene Karangozishvili Lafayette College
Advisor(s): Derek Smith, Lafayette College

The Sierpiński carpet and Menger sponge are well-studied generalizations of the Cantor set to higher dimensions, but they are only two of a doubly-infinite family of fractals that naturally generalize the Cantor set. Most of these higher-dimensional fractals are connected sets, so we study each with a goal of better understanding different types of paths that join two arbitrary points and stay in the fractal. In particular we determine the shortest taxicab distance between any two points in the fractal and construct an explicit path which realizes it. This allows us to compare the taxicab metric in the fractal with the standard Euclidean metric.

246. Intersections of Shortest Taxicab Paths in the Sierpiński Carpet

Rebekah Chase Evangel University
Ryan Mike CU Boulder
Laura Seaberg Haverford College
Advisor(s): Carl Hammarsten, Lafayette College

In recent work, Berkove and Smith have developed an algorithm to construct shortest taxicab paths in the Sierpiński carpet and some of its higher-dimensional generalizations. We consider an extension of this problem examining minimal area surfaces bound by shortest taxicab paths in higher-dimensional fractals. Such a minimal surface will have zero area if and only if the associated shortest paths have non-empty common intersection. Specifically, we give a set of necessary and sufficient conditions on the relative positions for three points in the carpet which characterize when the pairwise shortest taxicab paths have non-empty triple intersection. Finally, we indicate how our work might generalize to higher dimensions. This work was conducted as part of the Lafayette College Mathematics REU.

247. Equi-areal Clairaut Parametrizations of Surfaces in Real 3-Space

Elena Wang College of the Holy Cross
Advisor(s): Andrew Hwang, College of the Holy Cross

Gaussian curvature is a numerical invariant for surfaces introduced by C. F. Gauss in 1827. The Gaussian curvature quantifies the shape of a surface and is unchanged when the surface is bent without stretching. A surface with positive Gaussian curvature is convex; a surface with negative curvature is saddle-shaped and a surface with zero curvature is intrinsically flat. A general surface has the form $E(u,v)du^2 + F(u,v)(dudv + dvdu) + G(u,v)dv^2$. An equi-areal Clairaut parametrization has the form $G(u)^{-1}du^2 + G(u)dv^2$ and the Gaussian curvature is minus one-half the second derivative of G. The goal of the project was to classify geometrically accurate representations (isometric immersions) of equi-areal Clairaut parametrizations in real 3-space. This class of parametrizations is equipped with a natural orthonormal frame. The Cartan structure equation for this frame becomes a system of first-order partial differential equations, whose solutions correspond to isometric immersions.

248. A Vector Proof of de Guas Theorem

Jeffrey Wilkinson Washington & Jefferson College
Advisor(s): Roman Wong, Washington & Jefferson College

When the 3D Pythagorean Theorem is mentioned, most people think that it is about the square of the diagonal of a rectangular box being equal to the sum of the squares of the three sides. However, since the generalization of a right triangle in 2D is a trirectangular tetrahedron in 3D, the 3D extension of Pythagorean Theorem actually refers to the four faces of a trirectangular tetrahedron. De Guas theorem states that the square of the face opposite to the right angle corner of a trirectangular tetrahedron is equal to the sum of the square of each of the other three faces. The original theorem, which was published by Jean Paul de Gua de Malves in 1783, has been proven using a variety of methods over the years. Most of these many established proofs of de Guas theorem use the conventional Pythagorean Theorem or Herons area formula. However our new proof using vectors and cross product is simpler in algebra and is more symmetric than the other proofs. Our proof is a new and simple approach to a proof for a theorem more than two centuries old.

249. Relationship between Poincaré and Serre duality

Jose Lopez Iowa State University
Advisor(s): Michael Young, Iowa State University

This poster will be about the Poincaré duality and its relationship to the Serre duality. The concepts of the Poincaré lemma, Mayer-vietoris sequence, Kähler manifolds, and vector bundles will be introduced. Also, the poster will show examples of differential k-forms, de Rham Cohomology, and intuition behind Poincaré duality. One section of the poster will focus on Kähler manifolds, and how the Poincaré duality coincides with the Serre duality in the setting of Kähler structures. I believe Kähler manifolds are rich and detailed structures, and in this context they provide a connection between differential geometry and algebraic geometry.

250. All Tangled Up

Seth Colbert-Pollack Kenyon College
Micah Fisher Kenyon College
Advisor(s): Carol Schumacher, Kenyon College

A hypotrochoid (respectively epitrochoid) is drawn by tracing a point on a circle as it rolls inside (respectively around) another, fixed, circle. (Many people will recognize these curves as those drawn with a child's Spirograph toy.) We examine the symmetries of figures we call Tangloids which generalize hypotrochoids. They are drawn by tracing a point on a circle as it rolls inside (or around) more complicated curves called tangles. (Tangles are made up piecewise quarter circles, so a circle is the simplest tangle.) Our project investigates the rotation and reflection symmetries of tangloids. We found that a certain parameter predicts which tangloids have symmetries and which dont.

251. Asymptotics of a Group Action on a Character Variety

Seth Lee George Mason University
Julian Benali George Mason University
Advisor(s): Sean Lawton, George Mason University

The affine space $\mathbb{F}_q^3$, for q the power of a prime, can be partitioned into varieties defined by the zeroes of $\kappa(x, y, z) = x^2 + y^2 + z^2 - xyz - 2 - \lambda$ with λ in the base field. The morphisms $\iota(x, y, z) = (x, y, zy - x)$, $\eta(x, y, z) = (z, y, zy - x)$, and $\tau(x, y, z) = (y, x, z)$ preserve the κ-varieties and in fact generate a group we call Γ. We consider the group action of Γ on the κ-varietes which appears to act transitively in the limit as q tends to infinity. That is, the order of the largest orbit asymptotically approaches the cardinality of the κ-varieties. We pose conjectures on the structure of the orbits on these κ-varieties. Namely, that there are finitely many exceptional orbits (those whose order is bounded by a constant), and all orbits but one are of this type. We also present various methods used to produce partial results to the posed conjectures.

252. Efficiency of Planar Disk Packings

Rohit Banerjee UIC
Jacob Krol UIC
Advisor(s): Ali Mohajer, UIC

In any two-species packing of disks, let r be the ratio of the smaller radius to the larger. An open question in mathematics is to determine a sharp upper bound on the density of two-species packings in the plane as a function of r. This project seeks to obtain new information on this density function through the analysis of experimental data. We obtain data by generating finite packings in which the proportion of radii and the relative number of each species vary.

253. Differential Invariants of Curves and Surfaces in Lie Groups

Jordan Berkompas Longwood University
Advisor(s): Thomas Wears, Longwood University

In 1872, Felix Klein introduced his Erlangen program, which loosely states that a geometry is a set of geometric transformations (or symmetries) on a space of points. The study of a geometry then amounts to studying geometric objects in the space and identifying properties which remain invariant (or unchanged) under the given geometric transformations. In the spirit of Felix Klein's Erlangen program, we use the Fels-Olver method of moving frames to generate complete sets of differential invariants of curves and surfaces in the Lorentz-Minkowski plane and in select 3-dimensional Lie groups.

254. Creating a Database of Yang-Mills Solutions for the Differential Geometry Package in Maple

Ryan Bevan Utah Valley University
Eli Atkin Utah Valley University
Advisor(s): Alan Parry, Utah Valley University

Using non-Abelian Lie groups, Yang-Mills theory describes the behavior of elementary particles. This system of geometric PDEs successfully unifies the electromagnetic and weak forces and accurately describes quantum chromodynamics. Because of this, it is vital for understanding the standard model of particle physics.

The main goal of this project is to make solutions to the Yang-Mills equations easily accessible. We did this by creating a database of solutions and their properties. This database will be available as a free addition to the differential geometry package in Maple. Initially, the database will have 37 solutions from a review paper by Alfred Actor, but we plan to continue adding to this database in the future. The solutions are first input into Maple to find the connection one-form for each solution and add it to the database. Various properties presented in the paper for each solution are then tested and cataloged alongside the connection one-form. From the entries in the database, these Yang-Mills solutions can be called up in a format that can be manipulated in Maple lending a great deal of readily available computing power to studies about these solutions.

255. Exploring Maximum Proper Diameter of Graphs

Grant Fickes Kutztown University of Pennsylvania
Dylan Green Trevecca Nazarene University
Nathaniel Sauerberg Carleton College
Jill Stifano Fairfield University
Advisor(s): Karen McCready, King's College

In an edge-colored graph, a properly colored path is a path in which no two consecutive edges have the same color. A properly connected coloring of a graph is one in which there exists a properly colored path between every pair of vertices. Given a graph G with a properly connected coloring, the proper distance between any two vertices is the length of a shortest properly colored path between them. Furthermore, the proper diameter of G is the largest proper distance between any pair of vertices in G under the given coloring c. Since there can be many properly connected colorings of G, there are possibly many different values for the proper diameter of G. If G has n vertices, a natural upper bound for its proper diameter is $n-1$, but this value is not attainable for all graphs, such as graphs without a Hamiltonian path. We characterize 2-connected graphs for which this upper bound of proper diameter is attainable. This work was done at Lafayette College and was supported by the National Science Foundation under Grant No. 1560222.

256. Vertex Minimal Planar Graphs With Prescribed Automorphism Groups

Carlie Triplitt University of Science and Arts of Oklahoma
Sarah Lubow Loyola University New Olreans
Advisor(s): Lindsey Lauderdale, Towson University

Abstract. In 1939, Frucht proved that for any finite group G, there exists a graph Γ such that the automorphism group of Γ is isomorphic to G. Naturally, this result gave rise to numerous extremal problems in graph theory. For instance, vertex-minimal graphs with a prescribed automorphism group are the subject of prior research by numerous authors. In this talk, we will discuss our proof of a conjecture made in 1980 by Marušič on the order of vertex-minimal planar graphs with cyclic symmetry of even order. Our proof completes a theorem giving the order of all vertex-minimal planar graphs with cyclic automorphism groups. We will also discuss further our proof regarding the order of vertex-minimal planar graphs with dihedral symmetry. This work was completed as part of the REU program at University of Texas at Tyler.

257. Hall t-Chromatic Spectra and Weak Hall t-Chromatic Spectra of the Petersen Graph and of Wheels with Odd Numbers of Spokes

A. Sophia Aiken Colorado College
Advisor(s): Peter Johnson, Auburn University

A color demand function on a graph G is a function $\kappa : V(G) \to \mathbb{N}$. A proper (t,κ)-coloring of G is a function ϕ assigning each vertex of G a subset of $[t] = \{1, 2, , t\}$ so that for each $v \in V(G)$, $|\phi(v)| = \kappa(v)$ and for each $uv \in E(G)$, $\phi(u) \bigcap \phi(v) = \emptyset$. $\alpha(G)$ is the vertex independence number of G. G and κ satisfy Halls t-condition if and only if for each subgraph H of G,

$$t\alpha(H) \geq \sum_{v \in V(H)} \kappa(v).$$

It is clear that Halls t-condition is necessary for the existence of a proper (t,κ)-coloring of G. If it is sufficient (i.e. If G is properly (t,κ)-colorable for every color demand κ on G such that G and κ satisfy Halls t-condition) then G is Hall t-chromatic. If Halls t-condition with the equation $t\alpha(G) = \sum_{v \in V(G)} \kappa(v)$ suffice for the existence of a proper (t,κ)-coloring of G, then G is weakly Hall t-chromatic. We show that the Petersen graph is Hall 3-chromatic and determine the weak Hall t-chromaticity of wheels with odd numbers of spokes. Research from 2018 Auburn REU in Algebra and Discrete Math.

258. Minimal Embedding Dimensions of Rectangle k-Visibility Graphs

Espen Slettnes University of California, Berkeley
Advisor(s): Jesse Geneson, Iowa State University

Bar visibility graphs were adopted in the 1980s as a model to represent traces, e.g.. on circuit boards and in VLSI chip designs. Rectangle visibility graphs were introduced by Bose et al. in 1997 as a generalization of bar visibility graphs.

A graph is a rectangle visibility graph if it can be represented with vertices as disjoint axis-parallel rectangles such that there is an unobstructed axis-parallel line of sight between two rectangles if and only if there is an edge between the corresponding vertices. We combine rectangle visibility graphs with k-visibility to form rectangle k-visibility graphs, in which the line of sight between two rectangles in the representation are obstructed by at most k other rectangles. We then take a natural generalization of rectangle k-visibility graphs into higher dimensions. We find that given enough spacial dimensions and fixed k, there exists a rectangle k-visibility representation of any graph G, and find bounds on the smallest such dimension needed to represent various graphs. We continue to study its properties, and proceed to bound it for complete graphs, complete r-partite graphs, and hypercube graphs.

259. Hypergraphs with few Berge paths of fixed length between vertices

Zhiyang He Carnegie Mellon University
Advisor(s): Michael Tait, Carnegie Mellon University

In this paper we study the maximum number of hyperedges which may be in an r-uniform hypergraph under the restriction that no pair of vertices has more than t Berge paths of length k between them. When $r = t = 2$, this is the even-cycle problem asking for $\mathrm{ex}(n, C_{2k})$. We extend results of Füredi and Simonovits and of Conlon, who studied the problem when $r = 2$. In particular, we show that for fixed k and r, there is a constant t such that the maximum number of edges can be determined in order of magnitude.

260. Constructing Copoint Graphs of Convex Geometries

Sierra Knavel Ohio University
Giana Cirulli Eastern University
Advisor(s): Jonathan Beagley, Valparaiso University

We work with copoint graphs of convex geometries. Copoint graphs can be used to study the complex and fairly recent eld of convex geometries. Comparing copoint graphs and their convex geometries helps identify properties. We demonstrate that multiple convex geometries have the same underlying copoint graph. All graphs on one to ve vertices can be represented as possible copoint graphs of some convex geometry. Furthermore, we construct several innite classes of copoint graphs including the complete k-partite graph, path graph, centipede graph, ladder graph, comb graph, pom-pom graph, sharkteeth graph, and broken wheel graph. Completed during an REU at Valparaiso University.

261. Pattern Avoidance in Acyclic Digraphs

Meraiah Martinez Benedictine College
Otto Osterman University of Texas at Dallas
Xuming Liang Harvey Mudd College
Advisor(s): Christy Graves, University of Texas at Tyler

A pattern of length k is a permutation in S_k. A permutation $\pi \in S_n$ avoids the pattern σ if no subsequence of length k has elements in the same relative order as σ. Our research extends this concept to directed acyclic graphs. A directed acyclic graph G avoids the pattern if there are no directed paths whose vertices contain a subsequence in the same relative order as the pattern. For certain sets of length 3 patterns, we find the number of directed acyclic graphs on n vertices that avoid all of the patterns in the given set. Furthermore, for certain sets of patterns where exact enumerations have not been found, we provide a comparison to the number of graphs avoiding a different set of patterns. This work was completed as part of the REU program at the University of Texas at Tyler.

262. The coloring graph of complete graphs and Paley graphs

Haylee Harris CSU Fresno
Advisor(s): Oscar Vega, CSU Fresno

An n-coloring of a graph G is an assignment of colors to the vertices of G so that no pair of adjacent vertices are painted in the same color. Not all n colors need to be used in the coloring. The n-coloring graph of G is the graph with vertex-set the n-colorings of G, and with edges connecting n-colorings that differ at exactly one vertex of G. The main focus of this project is to study the structure of the n-coloring graph of complete graphs, and Paley graphs, for all n.

263. Failed Power Domination

Jonathan Tostado-Marquez Swarthmore College
Advisor(s): Cheryl Grood, Swarthmore College

A power dominating set of a graph G is a subset S of vertices of G for which application of the power domination process results in all vertices joining S. The first part of power domination process enlarges S by adding to it all of its neighbors: that is, if u is neighbor of a vertex in S, then u joins S. The next step of the process states that if there is a vertex v in S that has exactly one neighbor u not in S, then u joins S. This step is iterated until there are no more vertices in S with exactly one neighbor not in S. This process of adding certain vertices to an initial set of vertices S is modeled after the monitoring of electrical networks by phase measurement units (PMUs), where electric nodes correspond to vertices, edges represent transmission lines connecting the nodes, and PMUs are placed at the vertices initially in S.
We introduce a new graph parameter, the failed power domination number of a graph G, FP(G), to be the maximum cardinality of a subset S of vertices of G that is not a power dominating set. We determine formulas for the failed power domination number of several graph families and the Cartesian product of a cycle and P2.

264. Design Strategies for Modeling Ladder-based Graphs using DNA Self-Assembly

Jackson Hansen Lewis University
Tyler Starkus Lewis University
Advisor(s): Amanda Harsy, Lewis University

Motivated by the recent advancements in nanotechnology and the discovery of new laboratory techniques using the Watson-Crick complementary properties of DNA strands, formal graph theory has recently become useful in the study of self-assembling DNA complexes. In this poster, we present our results applying graph theoretical and linear algebra techniques for constructing tile-based self-assembling complexes including ladder graphs, Mongolian tent graphs, and square-based nanotubes graphs. We explore designs strategies in which graphs smaller than or equal to the target graphs are acceptable.

265. Using Graph Theory to Design Optimal Strategies for DNA Self-Assembly

Nick Soto Lewis University
Simon Merheb Lewis university
Advisor(s): Amanda Harsy, Lewis University

Motivated by the recent advancements in nanotechnology and the discovery of new laboratory techniques using the Watson-Crick complementary properties of DNA strands, formal graph theory has recently become useful in the study of self-assembling DNA complexes. Construction methods developed with concepts from undergraduate level graph theory have resulted in significantly increased efficiency. In this poster, we present our results of applying graph theoretical and linear algebra techniques to constructing bipartite graphs which can be created from self-assembling DNA. In particular, we present tile-based design strategies for constructing book and acyclic graphs in laboratory scenarios 1 and 2.

266. Modeling Cross-Prism and Petersen Graph Families in Self-Assembling DNA Using Graph Theory and Linear Algebra

Alvi Renzyl Cortes Lewis University
Lauren Gernes Lewis University
Eric Redmon Lewis University
Advisor(s): Amanda Harsy, Lewis University

Motivated by the recent advancements in nanotechnology and the discovery of new laboratory techniques using the Watson-Crick complementary properties of DNA strands, formal graph theory has recently become useful in the study of self-assembling DNA complexes. Construction methods based on graph theory have resulted in significantly increased efficiency. In this poster, we present the results of applying graph theoretical and linear algebra techniques for constructing graphs including cross-prism and Petersen graph families. In particular, we explore various design strategies given two laboratory constraints.

267. Coarse Ricci Curvature on Graphs

Conor Carroll California Polytechnic State University, San Luis Obispo
Uyen Dinh California Polytechnic State University, San Luis Obispo
Sydney Dye California Polytechnic State University, San Luis Obispo
Joshua Frederick California Polytechnic State University, San Luis Obispo
Advisor(s): Vincent Bonini, Califonia Polytechnic State University, San Luis Obispo

We study a modified notion of Olliviers coarse Ricci curvature on graphs introduced by Lin, Lu, and Yau. We show that a finite, undirected, simple graph is complete if and only if the Ricci curvature is strictly greater than one. In addition, we derive explicit formulae for the Ricci curvature of large classes of strongly regular graphs, which in particular show that all strongly regular graphs under consideration have positive Ricci curvature. We also extend some known estimates and properties of Ricci curvature to the modified and weighted settings. Lastly, we study the effect on the expected curvature values when one randomly adds or deletes an edge on circulant graphs.

268. Extremal Problems Related to the Cardinality Redundance of Graphs

Daniel McGinnis New College of Florida
Advisor(s): Nathan Shank, Moravian College

A dominating set of a graph G is a set of vertices D such that for all $v \in V(G)$, either $v \in D$ or $(v, d) \in E(G)$ for some $d \in D$. The cardinality redundance of a vertex set S, $CR(S)$, is the number of vertices in $V(G)$ such that $|N[x] \cap S| \geq 2$. The cardinality redundance of G is the minimum of $CR(S)$ taken over all dominating sets S. A set that achieves $CR(G)$ is a γ_{cr}-set, and the size of the minimum γ_{cr}-set is $\gamma_{cr}(G)$. We give the maximum number of edges in a graph with a given number of vertices and given cardinality redundance. In the cases that $CR(G) = 0$, 1, or 2, we give the minimum and maximum number of edges of graphs where $\gamma_{cr}(G)$ is fixed, and we give the minimum and maximum values of $\gamma_{cr}(G)$ when the number of edges are fixed. This work was completed at the Muhlenberg College REU program.

269. Nimbers of Node-kayles on Certain Families of Graphs

Riley Waechter Northern Arizona University
Advisor(s): Eugene Fiorini, Muhlenberg College

In this report we investigate the nimber values obtained from playing Node-Kayles on various families of graphs including lattices, linked complete graphs, and linked cycles. For some families of graphs, we are able to prove that Node-Kayles will be equivalent to a certain octal game. This work has resulted in multiple new sequences published on the OEIS. This work was done as part of an REU program at Muhlenberg College in Allentown, Pennsylvania.

270. Combinatorics of k-Farey Graphs

Miguel Lopez Boston University
Advisor(s): Jonah Gastor, McGill

With an eye towards studying curve systems on low-complexity surfaces, we introduce and analyze the k-Farey graphs $F_{<k}$ and $F_{\leq k}$, two natural variants of the Farey graph F in which we relax the edge condition to indicate intersection number $= k$ or $\leq k$, respectively. The former, $F_{<k}$, is disconnected when $k > 1$. In fact, we find that the number of connected components is infinite if and only if k is not a prime power. Moreover, we find that each component of $F_{<k}$ is an infinite-valence tree whenever k is even, and $\mathrm{Aut}(F_{<k})$ is uncountable for $k > 1$. As for $F_{\leq k}$, Agol obtained an upper bound of $1 + \min p : p$ is a prime $> k$ for both chromatic and clique numbers, and observed that this is an equality when k is either one or two less than a prime. We add to this list the values of k that are three less than a prime equivalent to 11 (mod 12), and we show computer-assisted computations of many values of k for which equality fails. This work was completed during the Summer@ICERM 2018: Low Dimensional Topology and Geometry Program.

271. Maximum efficiency, minimum effort: Fastest-mixing Markov chain on (m, n)-star graphs

Jacob Williams University of Wyoming
Advisor(s): Bryan Shader, University of Wyoming

Imagine a network of sensors in a star-like pattern, each collecting data at its particular location. It is desirable for these sensors to agree on a consensus value over the entire region, but if power consumption is a concern, this consensus

should be attained with the fewest possible communications. This problem may be attacked via the fastest-mixing Markov chain, or FMMC, on the network's associated graph. The FMMC is constructed for (m, n)-star graphs — that is, m paths of length n, each connected at one end to a central node — by minimizing the second-largest eigenvalue modulus (SLEM) of the associated Markov transition matrix. Optimal solutions are directly calculated for $n = 1$ and $n = 2$; for $n \geq 3$, the optimal solution is given by the spectral radius of a reduced $n \times n$ tridiagonal matrix T. Furthermore, with m fixed, the solution can be parametrized by a single variable, and the optimal solution is given by the solution of a system of two polynomials in that parameter. This research was supported by the Wyoming Research Scholars Program.

272. Arithmetical Structures on Complete Graphs

Zachary Harris Niagara University
Advisor(s): Joel Louwsma, Niagara University

An arithmetical structure on a finite graph is a pair $(\mathbf{d}, \mathbf{r})$ of vectors with positive integer entries such that $\mathbf{r}$ is primitive and $(D - A)\mathbf{r} = \mathbf{0}$, where D is the diagonal matrix with entries given by the vector $\mathbf{d}$ and A is the adjacency matrix of the graph. We study arithmetical structures on K_n, the complete graph on n vertices. We show that, for all positive integers k less than a certain value depending on n, there is an arithmetical structure on K_n for which the largest entry in $\mathbf{r}$ is k. We also show that, if k is a positive integer all of whose prime factors are greater than a certain value depending on n, there are no arithmetical structures on K_n for which the largest entry in $\mathbf{r}$ is k. In particular, if p is a prime number greater than a certain value depending on n, there are no arithmetical structures on K_n for which the largest entry of $\mathbf{r}$ is p.

273. Structure Theorem for Critical Groups of Iterated Cones of Graphs

Gopal Goel High school
Advisor(s): David Perkinson, Reed college

Let G be a finite connected graph. The nth iterated cone G_n of G is the join of G with the complete graph on n vertices. The critical group of a graph from divisor or sandpile theory is the torsion part of the cokernel of the graph's discrete Laplacian matrix. We give a structure theorem for the critical group of G_n.

274. Spectral properties of quaternion unit gain graphs

Christine Izyk The College at Brockport, State University of New York
Advisor(s): Nathan Reff, The College at Brockport, SUNY

Gain graphs are a special kind of graph where each orientation of an edge is given a group value, and the inverse of this group value is assigned to the opposite orientation. If these edge values are chosen to be unit quaternions, then we can define matrices associated to these graphs so that their algebraic properties can be related to the original graph. In particular, we can find the left and right-eigenvalues associated to these matrices and bound them using graph parameters. We define the adjacency, incidence and Laplacian matrices, and study each of them. We extend some fundamental concepts from spectral theory to this new setting.

275. Graph Theoretic Models of Interdependence in Referendum Elections

Colby Brown University of Arizona
Advisor(s): Jonathan Hodge, Grand Valley State University

In referendum elections, voters are often required to cast simultaneous votes on multiple questions. However, a voter's opinion on one question may depend on the outcome of another, and they must cast all their votes without knowing the outcome of certain questions. This may lead to undesirable or even paradoxical outcomes. When this occurs, we say that the questions were ”non-separable” in the minds of the voters. Determining characteristics of an election's separability may lead to the design of better elections, where the results more accurately reflect the will of the electorate. However, designing elections with a given separability is difficult and, in some cases, impossible. We introduce a new graph theoretic model to represent the separability of an election and show how this model may be used to answer many questions about separability while raising many new questions of its own. We will also consider how this model can be used to study alternative voting methods and strategies. This research was conducted at Grand Valley State University as part of their REU program, with support from the National Science Foundation.

276. Failed Zero Forcing on Oriented Graphs: Paths, Cycles and Other Results

Alyssa Adams Youngstown State University
Advisor(s): Bonnie Jacob, Rochester Institute of Technology

Let Λ be a directed graph with vertex set $V(\Lambda)$ and edge set $E(\Lambda)$ where $n = |V(\Lambda)|$ and each edge is assigned one or more orientations, $e = (u, v)$ or $e = (v, u)$. If $e = (v, u)$, then u is considered an out-neighbor of v. In zero forcing, each vertex is initially either filled or empty, then the color change rule (CCR) is applied, where the CCR is: if a filled vertex v has exactly one out-neighbor u, then u will be filled. The process continues until either all of the vertices in $V(\Lambda)$ are filled, or the CCR restricts the remaining empty vertices from being filled. If $V(\Lambda)$ cannot be completely filled, then the initial filled set of vertices is said to be a failed zero forcing set (FZFS). Specifically, we are looking for the failed zero forcing number $F(\Lambda)$ which is the maximum number of vertices in any FZFS. We can determine $F(\Lambda)$ for any weak path Λ. We also show that for any integer $n \geq 3$ and k where $0 \leq k \leq n-1$, there exists a weak cycle Λ such that $F(\Lambda) = k$. This work was done at the National Technical Institute for the Deaf at Rochester Institute of Technology for their 2018 REU program.

277. Reconfiguration Graphs: Exploring Networks of Solutions

Yitbarek Demesie La Salle University
Advisor(s): Janet Fierson, La Salle University

The goal of reconfiguration problems is to determine whether it is possible to transform one solution into another solution through a series of steps, or applications of a reconfiguration rule, in such a way that each intermediate step also results in a solution. This research explores reconfiguration graphs, structures used to study reconfiguration. We consider a graph problem (for example, dominating sets), in its original form with a natural reconfiguration rule, and then we adjust components of the reconfiguration process. For dominating sets, this includes changes to the size of solutions permitted, the reconfiguration rule (such as token addition/removal or token sliding), and the base graphs. After producing reconfiguration graphs under various combinations of conditions, we investigate the different themes that are exhibited and identify some resulting properties. We present results on which graphs may and may not appear as reconfiguration graphs or induced subgraphs of reconfiguration graphs for a given graph problem. In addition, we consider features of our reconfiguration graphs such as connectedness, order, and girth and other properties related to cycles.

278. Sandpiles on the several lattices

Hyojeong Son SUNY, Stony Brook University
Advisor(s): Robert Hough, SUNY, Stony Brook University

The abelian sandpile model, introduced by Danish physicist Per Back, was widely studied in physics to mathematics that has been practically applied to real-world scenarios such as modeling stock market or traffic jams. Suppose a set of vertices and edges is given and chips are distributed on each vertex. We call a vertex stable if it has a fewer number of chips than it's degree; otherwise, the vertex is unstable. We topple chips from one unstable vertex by sending out a chip to each neighboring vertex while setting one vertex as a sink, where passed chips to the sink are removed. In this project, we study sandpiles on the multiple lattices such as triangular, honeycomb, triakis-triangular lattices and even other lattices in higher dimensions. We develop Python code to analyze the spectral gap, the asymptotic mixing times, and the functions, which are harmonic modulo 1, on those lattices.

279. What's a DRACKN?

Alex Foster Coastal Carolina University
Advisor(s): James Solazzo, Coastal Carolina University

Equiangular Tight Frames, or ETFs, are sets of vectors that correspond to sets of maximal equiangular real (or complex) lines in Euclidean space. ETFs have sought after applications in various digital communication and coding theory contexts. This sparks an ongoing search to identify and classify ETFs. A common strategy to obtain new classes of ETFs is to utilize properties of certain graphs. Recently it has been shown that Distance Regular Antipodal Covers of K_n, or DRACKNs maintain certain properties can be used to construct new ETFs. Here we elaborate on these properties and demonstrate how these properties can be used to construct ETFs.

280. Swarms and Group Theory

Isabella Diaz St. Thomas Aquinas College
Advisor(s): Meghan DeWitt, St. Thomas Aquinas College

Group theory is used describe and predict countless events and swarms will be the next. Many of these biological groups exhibit behaviors that may be able to be connected by guidelines of algebraic group theory. These behaviors include a maximum and minimum distance between group members and a collective between them, such as direction of movement. These behaviors are currently being researched using differential equations. However, we believe that we are the first to study them using group theory. We investigate the group-like structure of swarms and seek to develop how these structures work with each other by taking advantage of the similarity to groups. The symmetrical tendencies behavioral patterns exhibit will be crucial in understanding the way swarms operate. Swarms often depend of environmental conditions and mate recognition with the largest density of individuals concentrated in the center of the swarm according to a 2009 study by Manoukis et. al.

281. Belted-Sum Decompositions of Fully Augmented Links

Brian Ransom Florida State University
Advisor(s): Rolland Trapp, California State University at San Bernardino

This project studied the factorization of Fully Augmented Links using the belted-sum decomposition operation - the inverse of the belted-sum operation as defined by Colin Adams. To do this, a correspondence between belted-sum decomposition in the link complement and in a fundamental region of its universal cover in Hyperbolic 3-Space was established. Judicious choice of fundamental regions in the universal cover illuminated that factorization terminates with 'prime' pieces but is not unique. Restrictions which can be placed on the factorization to restore uniqueness are currently being studied. This project began as part of the REU program at California State University at San Bernardino over the summer of 2018.

282. A New Upper Bound on $\eta(n)$

Gabriel Lopez Cal State San Bernardino
Advisor(s): Corey Dunn, Cal State San Bernardino

Algebraic curvature tensors are tools in differential geometry which we use to determine the curvature of a manifold. Some of these curvature tensors can be constructed from antisymmetric bilinear forms. The number of algebraic curvature tensors built from antisymmetric forms to linearly combine into any arbitrary algebraic curvature tensor in a vector space of dimension n is denoted $\eta(n)$. In this project we improve the upper bound on $\eta(n)$ when $n \geq 5$. This work was done as part of an REU at California State University, San Bernardino.

283. Piecewise Harmonic Finite Elements on Clough-Toucher Splits

Ashley Imus Towson University
Advisor(s): Tatyana Sorokina, Towson University

We study continuous piecewise harmonic polynomials (splines) in two variables defined over the Clough-Tocher split of a triangle. The Clough-Tocher split is obtained by choosing an interior point in the triangle (usually the barycenter), and then coning off to the three vertices of the original triangle. The continuous piecewise harmonic splines of a fixed polynomial degree form linear spaces. We prove dimension results for quadratic, cubic, and quartic splines. We also show how to construct Bernstein-Bezier bases for these spaces.

284. The Numerical Range of a Composition Operator on the Hardy Space

Laney Bowden Colorado State University
Julia Balukonis Providence College
Fatme Hourani University of Michigan - Dearborn
Ellie Lochner University of Wisconsin - Eau Claire
Advisor(s): John Clifford, University of Michigan - Dearborn

For a bounded operator T on a Hilbert Space $\mathbb{H}$, the numerical range of T is the subset $W(T)$ of $\mathbb{C}$ given by $W(T) = \{< Tx, x > : ||x|| = 1\}$. We study the numerical range of the composition operator, C_A, on the Hardy space $H^2(\mathbb{B}_n)$ where A is an $n \times n$ matrix that is a self-map of the unit ball. We show the set of homogeneous holomorphic polynomials

of degree k is a reducing subspace for C_A; it follows that $W(A) \subseteq W(C_A)$. In the special case where A is a weighted shift, $W(C_A) =$ convex hull$(W(A) \cup \{1\})$. We completely characterize the numerical range of the operator when A is unitarily similar to a Jordan-normal form that maps the ball to the ball by decomposing our operator into the direct sum of shifts and normal operators. This research was a result of the University of Michigan at Dearborn REU program in the summer of 2018.

285. Translating Calculus in the Physical World

Faith Hensley Marshall University
Advisor(s): Bonita Lawrence, Marshall University

Many students find concepts of calculus difficult to visualize and understand, so in an effort to resolve this disconnect, we have been investigating the use of a new teaching tool, a differential analyzer, as a vehicle for translating calculus in the physical world. A differential analyzer is a mechanical computer that uses wheel and disc mechanisms connected via gears to solve differential equations. There are valuable connections between written mathematics and the differential analyzers 3D model of mathematics which we believe may serve as a missing link in students comprehension of fundamental mathematical concepts. The focus of this presentation will be on explaining how a differential analyzer operates and offering a visual presentation of the way in which a differential analyzer models functions rates of change so that viewers can see for themselves the machines potential to positively impact mathematics education.

286. From PE to Math through Martial Arts

Allison Young Saint Joseph's University
Tetyana Berezovski Saint Joseph's University
Advisor(s): Tetyana Berezovski, Saint Joseph's University

In this study we created an interdisciplinary curriculum for upper middle/lower secondary mathematics classroom. The disciplines we engaged included mathematics, physical education, and martial arts. All activities are aligned with Physical Education, CCSSM and foundational elements in martial arts. The focus of the design was to engage students in real-world problem solving by applying the academic material they have learned to non-traditional academic tasks of higher cognitive demand. Students were asked to model martial arts moves using geometry and trigonometry in the dynamic geometry environment. Each animated investigation was supported by the set of mathematical questions, with challenging extensions that would go beyond grade level and exhibit interrelations between different grade levels and subject areas. The study shows that while performing martial arts elements in the gym, students can have meaningful mathematical experiences, ask deep questions and become better prepared physically and mathematically. If their body can, so can their brain!

287. Developing Proof-Driven Algorithms by Creating Mathematical Problem Sets

Elizabeth Wissler University of Central Oklahoma
Advisor(s): Kristi Karber, University of Central Oklahoma

Demand is rising for distance learning options, which has created a need for large libraries of problem sets. Creating these libraries manually is time-consuming and effort-intensive, which is an opportunity for automation. The availability of a virtually unlimited number of problem sets for a given topic gives teachers the ability to quickly create new example, homework, and test problems without the need to purchase pre-made problem sets. In this project, we propose a method to automate problem set generation for distance learning. Floyd-Hoare logic is a systematic technique that is used to determine the correctness of an algorithm. Problem sets can be deconstructed and reverse-engineered into an abstraction of the concepts the problems represent. Using Floyd-Hoare logic, the abstraction can then be written simultaneously as a mathematical proof and an algorithm which acts as a constructor function for the problem sets.

288. GeoGebra

Rachel Gammal Emmanuel College
Advisor(s): Heather Pierce, Emmanuel College

This summer, we created and published a series of applets in Geogebra designed to provide visual aids to students enrolled in College Geometry, as well as to provide a free resource for public education. The primary applets we con-

structed modeled spherical geometry, the Half-Plane Model for hyperbolic space, and the Poincare Disk for hyperbolic space.

289. Formulas for Chebotarev densities of Galois extensions of number fields

Katharine Woo Stanford University
Naomi Sweeting University of Chicago
Advisor(s): Ken Ono, Emory University

We generalize the Chebotarev density formulas of Dawsey (2017) and Alladi (1977) to the setting of arbitrary finite Galois extensions of number fields L/K. In particular, if $C \subset G = \mathrm{Gal}(L/K)$ is a conjugacy class, then we establish that the Chebotarev density is the following limit of partial sums of ideals of K:

$$-\lim_{X\to\infty} \sum_{\substack{2\le N(I)\le X \\ I\in S(L/K;C)}} \frac{\mu_K(I)}{N(I)} = \frac{|C|}{|G|},$$

where $\mu_K(I)$ denotes the generalized Möbius function and $S(L/K;C)$ is the set of ideals $I \subset \mathcal{O}_K$ such that I has a unique prime divisor $\mathfrak{p}$ of minimal norm and the Artin symbol $\left[\frac{L/K}{\mathfrak{p}}\right]$ is C. To obtain this formula, we generalize several results from classical analytic number theory, as well as Alladi's concept of duality for minimal and maximal prime divisors, to the setting of ideals in number fields. This work was done at the REU program at Emory University under the direction of Ken Ono.

290. A Formula for the Number of Monic Degree m Polynomials in $\mathbb{F}_q[x]$ with Discriminant d

Michael Seaman Caltech
Advisor(s): Zavosh Amir-Khosravi, Caltech

We show a formula for the distribution of discriminants of monic polynomials over a finite field. For an odd prime power q, integer $m \ge 2$, and $d \in \mathbb{F}_q$, let $|V_d^m(\mathbb{F}_q)|$ be the number of monic polynomials in $\mathbb{F}_q[x]$ of degree m with discriminant d. We express $|V_d^m(\mathbb{F}_q)|$ as a discrete Fourier transform of Gauss sums, computable in polynomial time. For $d \neq 0$, we show

$$|V_d^m(\mathbb{F}_q)| = \chi(d) \sum_{c=1}^{q-1} \frac{G_{\mathbb{F}_q}(c)^m \frac{qB_{m-1}(c)-B_m(c)}{q} \tau_q(-1)^{\frac{cm(m-1)}{2}} \tau_q(d)^c}{G_{\mathbb{F}_q}(cm)}$$

where τ_q is a multiplicative character of order $q-1$, ψ a nontrivial additive character, $G_{\mathbb{F}_q}(c)$ is the Gauss sum $G_{\mathbb{F}_q}(\tau_q^c, \psi)$, χ is the quadratic character, and

$$B_k(c) = \begin{cases} q^{\frac{k \gcd(c,q-1)}{q-1}}, & \text{if } (q-1)|ck \\ 0, & \text{otherwise} \end{cases}$$

Classical and new discriminant distribution properties are shown as corollaries.

291. Eta-quotients of Prime or Semiprime Level and Elliptic Curves

Asimina Hamakiotes Macaulay Honors at Baruch College
Advisor(s): Holly Swisher, Oregon State University

In this paper, we investigate questions related to eta-quotients. We prove that eta-quotients that satisfy a condition of Newman are always modular forms if N is coprime to 6, and study the case of all odd levels. We also prove that all eta-quotients which are modular forms for $\Gamma_1(N)$ are also modular forms for $\Gamma_0(N)$. We found a condition for the existence of an eta-quotient as a modular form for prime levels, and generalized this for semiprime levels as well, which has improved upon recent work of Arnold-Roksandich, James, and Keaton. In addition, we investigate representing modular forms associated to elliptic curves in terms of linear combinations of eta-quotients, and provide some new examples. This work was done at an REU at Oregon State University.

292. On the Characterization of $\tau_{(n)}$-Atoms

Andre Hernandez-Espiet University of Puerto Rico - Mayaguez
Advisor(s): Reyes Ortiz-Albino, University of Puerto Rico - Mayaguez

In 2006, Anderson and Frazier define the concept of $\tau_{(n)}$-factorization, where $\tau_{(n)}$ is a restriction of the modulo n equivalence relation. These relations have been worked a lot for small values of n. However, it is sometimes difficult to extend findings to larger values of n. One of these problems is finding $\tau_{(n)}$-irreducible elements or $\tau_{(n)}$-atoms in order to characterize elements that have a $\tau_{(n)}$-factorization in $\tau_{(n)}$-atoms. The $\tau_{(n)}$-irreducible elements are well known for $n = 0, 1, 2, 3, 4, 5, 6, 8, 10, 12$. However, the problem of determining the $\tau_{(n)}$-atoms becomes much more difficult the larger n is. In this work, we present an easy way to construct families of $\tau_{(n)}$-atoms, where n is a safe prime associated with a Germain prime.

293. Index Divisibility of Dynamically Generated Sequences

Annie Schenck Mount Holyoke College
Advisor(s): Margaret Robinson, Mount Holyoke College

The index divisibility set of a sequence (s_n) is the set of positive integers n for which $n \mid s_n$. In a recent paper, Chen, Gassert and Stange characterize the index divisibility set for the sequence $(\phi^n(0))$, where $\phi(x) = x^d + c$ is an integral polynomial of degree at least two. Motivated by these results, we consider nonmonic polynomials. In particular, we describe the index divisibility set for the orbit of zero and give a visualization of the set as a directed graph. This work is continued from an REU at Hobart and William Smith Colleges supported by the National Science Foundation under grant no. DMS 1757616.

294. Exact Formulas for Invariants of Hilbert Schemes

Nate Gillman Wesleyan University
Matthew Schoenbauer University of Notre Dame
Advisor(s): Ken Ono, Emory University

A theorem of Göttsche establishes a connection between cohomological invariants of a complex projective surface S and corresponding invariants of the Hilbert scheme of n points on S. This relationship is encoded in certain infinite product q-series which are essentially modular forms. Here we make use of the circle method to arrive at exact formulas for certain specializations of these q-series, yielding convergent series for the signature and Euler characteristic of these Hilbert schemes. We also analyze the asymptotic and distributional properties of the q-series' coefficients. This work was carried out during the 2018 Research Experience for Undergraduates at Emory University.

295. p-adic Properties of Hauptmoduln with Applications to Moonshine

Ryan Chen Princeton University
Samuel Marks Princeton University
Matthew Tyler Princeton University
Advisor(s): Ken Ono, Emory University

The theory of monstrous moonshine asserts that the coefficients of Hauptmoduln, including the j-function, coincide precisely with the graded characters of the monster module, an infinite-dimensional graded representation of the monster group. On the other hand, Lehner and Atkin proved that the coefficients of the j-function satisfy congruences modulo p^n for $p \in \{2, 3, 5, 7, 11\}$, which led to the theory of p-adic modular forms. We combine these two aspects of the j-function to give a general theory of congruences modulo powers of primes satisfied by the Hauptmoduln appearing in monstrous moonshine. We prove that many of these Hauptmoduln satisfy such congruences, and we exhibit a relationship between these congruences and the group structure of the Monster. We also find a distinguished class of subgroups of the Monster with graded characters satisfying such congruences. This research was completed as part of the Mathematics REU at Emory University.

296. Perfect and Deficient Perfect Numbers

Emily Rachfal Kenyon College
Advisor(s): Judy Holdener, Kenyon College

The study of perfect numbers has spawned the definition and investigation of several related numbers: almost perfect numbers, deficient perfect numbers, and spoof perfect numbers. At first glance, these perfect spin-offs might appear

to be curious alternatives to perfect numbers, arising out of the intractability of problems related to perfect numbers. However, a closer look reveals that these perfect variants are inextricably linked to perfect numbers. If n is a positive integer, then $\sigma(n)$ denotes the sum of the divisors of n. The number n is said to be *deficient perfect* if $\sigma(n)/n = (2x - 1)/x$ for some $x \in \mathbb{N}$. We show that every perfect number has a deficient perfect divisor, generalizing a consequence of Euclid and Euler's characterization of even perfect numbers, namely that every even perfect number has an almost perfect divisor.

297. The Supersingularity of Hurwitz Curves

Michael Lynch Colorado State University
Seamus Somerstep Colorado State University
Advisor(s): Rachel Pries, Colorado State University

The first supersingular curves found were supersingular elliptic curves. Supersingular curves are useful in error-correcting codes. They also have potential applications to quantum resistant cryptosystems. Here we study when Hurwitz curves are supersingular. Specifically, we show that the curve $H_{n,\ell} : X^nY^\ell + Y^nZ^\ell + Z^nX^\ell = 0$, with n and ℓ relatively prime, is supersingular over the finite field $\mathbb{F}_p$ if and only if there exists an integer i such that $p^i \equiv -1 \bmod (n^2 - n\ell + \ell^2)$. If this holds, we prove that it is also true that the curve is maximal over $\mathbb{F}_{p^{2i}}$. Further, we provide a complete table of supersingular Hurwitz curves of genus less than 5 for characteristic less than 37. This research was part of an REU program with faculty advisor Dr. Rachel Pries and graduate advisor Dean Bisogno. The research team was composed of Erin Dawson, Henry Frauenhoff, Michael Lynch, Amethyst Price, Seamus Somerstep, and Eric Work.

298. Continued Fractions

Alexander Boettger Morningside College
Jared Amundson Morningside College
Advisor(s): Chris Spicer, Morningside College

This poster covers our study of continued fractions. Continued fractions, among other things, give us a way to represent irrational numbers with a rational approximation. We consider several theorems and properties related to the period length of irrational numbers and representations of all real numbers as continued fractions. In our research, we used the period length of infinite continued fractions to look for certain trends with a variety of different numbers, such as the square root of even and odd numbers, to characterize sets of numbers.

299. Modeling Discrete Time Series Using Lucas Numbers

Cheyenne Petzold Montclair State University
Advisor(s): Aihua Li, Montclair State University

This research focuses on discrete time series made by Lucas numbers. The time series we consider is made by n points, each of them are made of m consecutive Lucas numbers. Our main goal use linear algebra methods to construct polynomial models for such time series to study the properties of our models. Finding a model is equivalent to solving a matrix equation, where the associate matrix consists of the time series points. After appropriate row reductions, the resulting matrix may have good patterns such as a relationship to the Fibonacci numbers. A set of linear solutions was found. For future research, we will apply desired constraints to our model, explore methods for constructing nonlinear models, and study properties of constructed models.

300. First Moment of Quadratic L-functions in Function Fields

Dona Pantova Macalester College
Advisor(s): Ian Whitehead, Macalester College

Quadratic L-functions are a family of series which generalize the Riemann zeta function. Each L-function can be written as a sum over the integers, and has a line of symmetry at $\mathrm{Re}(s) = 1/2$.The coefficients of the series are given by Jacobi symbols modulo a fixed n L-functions have analogs in the rational function field, where the Jacobi symbol of m modulo n is replaced with the Jacobi symbol of a polynomial $f(x)$ modulo a fixed $g(x)$. We explore the growth of the family of quadratic L-series in the function field. In order to understand how fast this family of L-functions grows, we study the average of the series over all monic square-free polynomials g of a fixed degree.

301. On the Radical of Multiperfect Numbers

Viraj Jayam none
Carrie Zhang none
Ajit Kadaveru none
Advisor(s): Oleksiy Klurman, Royal Institute of Technology (KTH)

We consider how the radical of a multiperfect number is bounded relative to the multiperfect number. Previously, the radical of a perfect number has been bounded similarly. The goal is to find the smallest β such that $rad(m) < m^\beta$ for any multiperfect number m. It is conjectured that $\beta = \frac{1}{2}$. We obtain different values for β depending on the parity of the abundancy of the multiperfect number and the parity of the number itself, including $\beta = \frac{1}{2}$ when both the abundancy and the multiperfect number are odd. Furthermore, we show that the ABC Conjecture implies that only finitely many solutions to diophantine equations involving multiperfect numbers and factorials exist, as well as finitely many possibilities for the difference between perfect and multiperfect numbers. We also show that the ABC Conjecture implies that polynomials with large degree without repeated roots cannot be equal to infinitely many k-perfect numbers for any k. Finally, we prove that there are only finitely many multiperfect multirepdigit numbers in any base g where the number of digits in the repdigit is a power of 2.

302. Asymptotic Bounds for Extended Elliptic Pseudoprimes

Dylan Fillmore University of South Carolina
Alice Lin Princeton University
Philip Lamkin Carnegie Mellon University
Calvin Yost-Wolff Massachusetts Institute of Technology
Advisor(s): Liljana Babinkostova, Boise State University

Efficiently distinguishing prime numbers is a fundamental problem in mathematics. Many primality tests are built on this principle: From an input number N, define an abelian group G_N and test whether an element $x \in G_N$ acts as if N were prime. A composite number N is called a pseudoprime for $x \in G_N$ if it passes the primality test for x, and N is called a Carmichael number if it is a pseudoprime for all $x \in G_N$. In 1987, Gordon introduced pseudoprimes for CM elliptic curves. In 2012, Silverman extended this notion to all elliptic curves, and in 2017, Babinkostova et al. further defined strong elliptic pseudoprimes. Using techniques from analytic number theory, we improve bounds given by Schlage-Puchta for the probability that a fixed N is an elliptic Carmichael number for a random curve either when the largest prime factor p exactly dividing N is sufficiently large, or when $\omega(N)$, the number of distinct prime factors of N, is sufficiently large. Furthermore, we show the probability N is a strong elliptic Carmichael number is $\mathcal{O}(\log(\omega(N))/3^{\omega(N)})$. This research was conducted at an REU at Boise State University.

303. Explicit Sato-Tate for Primes in Arithmetic Progressions

Casimir Kothari Princeton University
Trajan Hammonds Carnegie Mellon University
Hunter Wieman Williams College
Advisor(s): Steven Miller, Williams College

Let $\tau(n)$ be Ramanujan's tau function, defined by

$$q \prod_{j=1}^{\infty} (1 - q^j)^{24} = \sum_{n=1}^{\infty} \tau(n) q^n;$$

(this is the unique holomorphic normalized cuspidal newform of weight 12 and level 1). Lehmer's conjecture asserts that $\tau(n) \neq 0$ for all $n \geq 1$; this is equivalent to $\tau(p) \neq 0$ for all primes p. Assuming standard conjectures for the twisted symmetric power L-functions associated to τ (including GRH), we prove that if $x \geq 10^50$, then

$$\#\{x < p \leq 2x : \tau(p) = 0\} \leq 5.973 \times 10^{-7} \frac{x^{3/4}}{\sqrt{\log x}}.$$

This is a consequence of an explicit form of the Sato-Tate conjecture (under the aforementioned conjectures), refined for primes in an arithmetic progression. This work was done at the SMALL REU at Williams College, under advisors Steven J. Miller and Jesse Thorner.

304. Parameters of locally recoverable codes with multiple recovery sets

Sam Kottler Colorado College
Advisor(s): Beth Malmskog, Colorado College

A code is a set of vectors, called codewords. Usually we look at codes that actually form vector spaces. Codes can be used for redundancy and error correction, when storing or transferring data. One way to do this is with locally recoverable codes (LRCs) in which any position of a codeword can be recovered from a fixed subset of other positions, called a recovery set. An interesting problem is called the availability problem, which addresses constructing LRCs with multiple disjoint recovery sets for each position. This project studied minimum distance and other parameters of families of such codes constructed from curves over finite fields.

305. Bitcoin, Blockchain Technology and Secure Hash Algorithms

Daniel Plummer Howard University
Advisor(s): Larry Washington, University of Maryland

In 2007, Satoshi Nakamoto published a paper titled, Bitcoin: A Peer-to-Peer Electronic Cash System. The concept created electronic cash by using digital signatures and proof-of-work. We define an electronic coin as a chain of digital signatures. Each owner transfers the coin to the next by digitally signing a hash of the previous transaction and the public key of the next owner and adding these to the end of the coin. [Satoshi Nakamoto 2007] In 2017, Google and CWI-Amsterdam used cloud computing to find a collision for the SHA-1 algorithm. This attack required over 9,223,372,036,854,775,808 SHA-1 computations, the research team said. This took the equivalent processing power of 6,500 years of single-CPU computations and 110 years of single-GPU computations. [https://shattered.io/static/shattered.pdf] Theoretically, a blockchain using SHA-1 would prove vulnerable to transaction altering and integrity from two different messages producing the same hash value making blockchain and cryptocurrencies infeasible. We argue that SHA-3 and SHA-256 still provide reliable and secure algorithms in a blockchain system.

306. Generalizing the Abundancy of an Integer

David Luo Emory University
Advisor(s): David Zureick-Brown, Emory University

The abundancy index of a positive integer n is the ratio of the sum of its divisors to itself; the abundancy index of n is two if and only if n is perfect. An abundancy outlaw is a rational number greater than one that fails to be the abundancy index of any positive integer. In this presentation, we generalize previous results about abundancy outlaws by defining a two variable abundancy index function as $I(x,n)\colon \mathbb{Z}^+ \times \mathbb{Z}^+ \to \mathbb{Q}$ where $I(x,n) = \frac{\sum_{d|n} d^x}{n^x}$. By exploring upper bound properties of the abundancy index, we construct sufficient conditions for rationals greater than one that fail to be in the image of $I(x,n)$. Finally, we apply these results to observe properties of perfect numbers under the two variable abundancy index.

307. On Generalizations of Sheldon Primes

Clare Kortlever Morningside College
David Swerev Morningside College
Advisor(s): Chris Spicer, Morningside College

In a recent Math Horizons article, the authors introduced the concept of a Sheldon prime. In this poster, we extend and generalize their results to different bases, and consider several variations of the product property introduced.

308. Effective Bounds for Traces of Maass-Poincaré Series

Havi Ellers Harvey Mudd College
Meagan Kenney Bard College
Advisor(s): Riad Masri, Texas A&M University

Works of Duke/Jenkins and Miller/Pixton show that the generating functions of traces of Maass-Poincaré series appear as holomorphic parts of certain half-integral weight weakly holomorphic modular forms. In this poster, we will present an effective upper bound for traces of Maass-Poincaré series, which can then be used to give effective upper bounds

for Fourier coefficients of these half-integral weight modular forms. This research was conducted as part of an REU at Texas A&M University.

309. Sums of Two Polygonal Numbers in Rings

Hongkwon Yi University of California, Berkeley
Advisor(s): Joshua Harrington, Cedar Crest College

In 1640, Fermat wrote a letter to Mersenne regarding a question about when a natural number can be expressed as a sum of two squares. Thanks to Euler, we fully understand the answer to this question in $\mathbb{Z}$. April 2nd 2014, Harrington, Jones, and Lamarche published a paper that explains sufficient and necessary set of conditions for when every element in the ring $\mathbb{Z}_n$ can be expressed as a sum of two squares. Our research motivates from the realization that square numbers are just a specific type of polygonal number; namely 4-gonal numbers. We carried out our research with the goal of finding the sufficient and necessary set of conditions for when every element in $\mathbb{Z}_{n\geq 2}$ can be expressed as a sum of two s-gonal numbers while (1) allowing and (2) not allowing zero as a summand. Most of the work was first done in $\mathbb{Z}_{p^\alpha}$, with prime p, using tools in algebraic number theory such as properties of quadratic residues; modular arithmetic; and also combinatorics. Then by applying the Chinese Remainder Theorem, we were able to geneate relevant conditions for $\mathbb{Z}_n$.

310. Jointly Ranked Prime-Reciprocal Sums and Natural Logarithms

Cahron Cross Prairie State College
Advisor(s): Steve Kifowit, Prairie State College

The aim of the research is to investigate the sequence generated by jointly ranking prime reciprocal sums up to the nth prime with ln(ln(n)). The location of the prime-reciprocal sums determine the numbers in the sequence. The objectives are to derive a formula for it and examine its characteristics, namely the differences between consecutive terms, and the differences between said differences. They will be referred to as first differences and second differences. The first differences were proven to tend to infinity. The distribution of the second differences showed interesting patterns, and it was proven that, after the first term, they could never have an absolute value greater than 2.

311. The Structure of 'Circular Farey Series'

Lee Trent Rose-Hulman Institute of Technology
Advisor(s): Timothy All, Rose-Hulman Institute of Technology

The Farey sequence of order n is the increasing sequence of reduced rational numbers between 0 and 1 inclusive whose denominator is at most n. While these sequences exhibit lots of very simple and intuitive properties due to their symmetry and order, they also exhibit more subtle and intricate properties. In particular, in 1946, Neville showed that the density of the rational numbers in the Farey sequence of order n is roughly uniform on the interval between 0 and 1 inclusive. We have examined an analogous sequence of rational points on the unit circle, which seem to exhibit similar patterns and underlying structures, using analogous techniques. This work could be further generalized to describe the behavior of the distributions of rational points along other curves.

312. New Theorems for the Digraphs of Commutative Rings

Morgan Bounds Indiana Wesleyan University
Advisor(s): Melvin Royer, Indiana Wesleyan University

The digraphs of commutative rings under modular arithmetic reveal intriguing cycle patterns, many of which have yet to be explained. To help illuminate these patterns, we establish a set of new theorems. Rings with relatively prime moduli a and b are used to predict cycles in the digraph of the ring with modulus ab. Rings that use Pythagorean primes as their modulus are shown to always have a cycle in common. Rings with perfect square moduli have cycles that relate to their square root.

313. Pascal's Triangles

Julius Premdas Cal Poly Pomona
Advisor(s): Jennifer Switkes, Cal Poly Pomona

The way we read or visualize numbers has changed time and time again. Through the discovery of the number 0, the discovery of rational and irrational numbers, and the discovery of complex numbers. We are not introducing new

numbers but we are introducing a new way numbers behave and can be read or visualized. We begin visualizing the rows of Pascals Triangle under a new method across dimensions. The new method is a mixture of concatenation, number base behavior, and the Carry Over Method. Then we will explore other triangles similar to Pascals Triangle, apply the new method across dimensions, and discover their behavior. In the end we will explore the behavior with all numbers and prime numbers unique behavior.

314. An Adaptive, Highly Accurate and Efficient, Parker-Sochacki Algorithm for Numerical Solution to Large Scale Dynamical Systems

Jenna Guenther James Madison University
Morgan Wolf James Madison University
Advisor(s): Paul Warne, James Madison University

For nonlinear systems of differential equations, an explicit adaptive procedure using a foundation of the Parker-Sochacki Method (PSM) produces better accuracy in less time with significantly fewer steps when contrasted with standard adaptive algorithms with a Runge-Kutta (RK) foundation. Two simple PSM functions are developed, illustrating a class of functions that represent the basis of a future PSM tool for the scientific community. At each step across the domain, combinations of these functions efficiently and recursively generate the coefficients of the Taylor polynomial of the solution to the ODE system. An adaptive stepping algorithm is derived, providing a simple way to increase/decrease the order of the method during computation. PSM Adaptive is developed theoretically and demonstrated on examples, including a 2 degree of freedom system related to missile defense. Results are compared to standard RK adaptive algorithms including 4th/5th order Dormand Prince, the foundational algorithm of MATLABs ODE45 solver. Noted in the 2 degree of freedom example, PSM Adaptive takes roughly two orders of magnitude fewer steps and runs an order of magnitude faster for similar accuracy.

315. Spectrum-Adapted Polynomial Approximation for Matrix Functions

Li Fan Macalester College
Advisor(s): David Shuman, Macalester College

Computations of the form $f(\mathbf{A})\mathbf{b}$ play an important role in applications in signal processing, machine learning, applied mathematics, and other disciplines. We propose and investigate two new methods to approximate $f(\mathbf{A})\mathbf{b}$ for large, sparse, Hermitian matrices $\mathbf{A}$. The main idea behind both methods is to first estimate the spectral density of $\mathbf{A}$, and then find polynomials of a fixed order that better approximate the function f on areas of the spectrum with a higher density of eigenvalues. Compared to state-of-the-art methods such as the Lanczos method and truncated Chebyshev expansion, the proposed methods tend to provide more accurate approximations of $f(\mathbf{A})\mathbf{b}$ at lower polynomial orders, and for matrices $\mathbf{A}$ with a large number of distinct interior eigenvalues and a small spectral width.

316. Efficiency of a Moving Mesh System with a Curvature-type Monitor and an Application to Burgers' Equation

Annaliese Keiser Bowling Green State University
Marianne Debrito Lawrence Technological University
Taima Younes University of Michigan-Dearborn
Advisor(s): Joan Remski, University of Michigan-Dearborn

Moving Mesh Methods are adaptive techniques to approximate solutions to partial differential equations numerically. A MM system consists of a physical PDE that evolves in time together with a PDE that adapts the discretization mesh using a monitor function. We explore properties of the MM system when the physical solution has a steep gradient and large curvature, depending on parameter ϵ, over a small interval in the domain. Using a curvature-type monitor, we prove an explicit dependence of the derivatives of mappings between the computational and physical domains on ϵ. In addition, we show a similar dependence for the mesh spacing, which quantifies discretization errors. Numerical evidence verifies these results and also suggests a significant reduction in steep gradients when using this type of monitor. These estimates show an explicit reduction of the number of equations needed to approximate the physical PDE with the moving mesh. We further control mesh spacing and steepness of derivatives by adjusting parameters in the monitor function. As an application, we use our moving mesh system to model Burgers' Equation, which satisfies the hypothesis of our theorem.

317. New Criteria for Comparing Global Stochastic Derivative-Free Optimization Algorithms

Jonathan McCart SUNY Geneseo
Advisor(s): Ahmad Almomani, SUNY Geneseo

For many situations, the function that best models a situation or data set can have a derivative that may be difficult or impossible to find. Thus, numerical methods for finding these important values without the direct involvement of the derivative have been developed to find the optimal value of the function. This is our motivation to use Derivative-free optimization (DFO) algorithms. In our analysis of these algorithms, we tested three global solvers: Genetic Algorithm (GA), Particle Swarm Optimization (PSO), and Simulating Annealing (SA) on a set of 25 problems of varying types: convex/non-convex, separable/non-separable, differentiable/non-differentiable, and unimodal/multimodal. For each algorithm, we used the built-in code from MATLAB, unedited or revised. For all problems, we varied the number of dimensions, increasing from 2 dimensions to 100 dimensions. We introduce new criteria to compare DFO solver performance using certain generalized characteristics that depend on speed and efficiency. Numerical results proposed for most known standard benchmark problems.

318. An Analysis of the Profitability and Volatility of Bitcoin and Standard Money

Kevin Womack Morehouse College
Advisor(s): Shelby Wilson, Morehouse College

Since the creation of Bitcoin, the first cryptocurrency, in 2008 by Satoshi Nakamoto, a number of cryptocurrencies have appeared on the market. One property of all such currencies is that their prices are considerably volatile. Early investors have obtained significant profits. We aim to answer whether or not Bitcoin and its counterparts will continue to be profitable. Here we use methods from the field of Numerical function approximation (least squares, Fourier Transforms, moving averages) to make predictions. After collecting the daily closing prices for a number of cryptocurrencies the year of 2017, we examine the stability versus profitability of these currencies as well as compare these results to corresponding results for standard money.

319. Fast Convergence with Series Expansion in Riemann Zeta Function

Ho Lung Tsui Exeter College, University of Oxford
Advisor(s): Robert Van Gorder, University of Oxford

Flajolet and Vardi (1996) proposed a method of representing certain infinite series as series of Riemann zeta function values through a change of basis formula. In this poster, we discuss their work, and establish results on the convergence properties of such series. We then present novel infinite series and infinite product representations for several mathematical constants in terms of Riemann zeta function values, and demonstrate that such representations converge more rapidly than do more standard representations.

320. A Mathematical Model of Maximizing Matching Rate Between Students and Advisors

Hanmi Zou College of William and Mary
Chengwu Shen College of William and Mary
Advisor(s): Anke Van Zuylen, College of William and Mary

Operational Research focuses on how to make an effective mathematical model that is useful for the allocation of limited resources, such as material, machines etc., to several competing activities, thus maximizing the optimal solution. People usually use the operational model to improve the decision-making process. In this project, we will demonstrate how to use the linear operational model, specifically, the transportation problem algorithm to optimize the matching between freshman and pre-major advisors, and to maximize the satisfaction degree between these two groups. The project will give generally ideas of background of operational research, details of transportation problem algorithm, and our own modified model based on the transportation problem algorithm.

321. Intelligently Segmenting the Long Tail

Carley Maupin Lewis University
Advisor(s): Amanda Harsy, Lewis University

An application of new algorithms that identify subpopulations using k-Disjunctive Normal Forms (k-DNFs), for which regression provides low-error predictions is being developed to investigate the quality of models produced and overall

proportion of the population covered by the discovered segments on some real-world domain, as compared to standard clustering techniques. Our research group worked with the Brown School of Social Work at Washington University in Saint Louis as a part of the Relationship and Sexual Violence Assessment Initiative (RSV-AI), which is developing an assessment system to address relationship and sexual violence. The initiatives goal is to understand how to effectively allocate treatment services to students, especially for those who may have less access to treatment or who may not generally seek treatment. Our research group is assisting the RSV-AI in seeking to identify which subpopulations benefit from treatment, and we are aiming to understand the effectiveness of treatment at different time periods. This project was a part of the Computer Science and Engineering REU at Washington University in St. Louis.

322. Multilevel Simulation of Stochastic (Local) Volatility Model and Applications

Michael Johnson Marist College
Casey Schoeller Marist College
Advisor(s): Duy Nguyen, Marist College

Stochastic (local) volatility models are ubiquitous in finance as they provide important alternatives for diffusion models such as the Geometric Brownian Motion model, which models asset prices such as stock price. Representative models are those of Heston, Scott, Hull-White, Stein-Stein, 3/2, SARB and recently proposed models such as alpha-Hypergeometric, 4/2, and Jacobi models. However, it is difficult (if not impossible) to simulate stochastic models exactly. Hence, numerical approximation is considered. In this poster, we consider a class of stochastic (local) volatility models which nest models listed above a special case. We employ the multilevel Monte Carlo proposed in Giles (2008) to simulate them numerically. Extensive numerical examples across a wide range of models are provided for demonstration and comparison.

323. How Healthy is Rhode Island: Life Satisfaction and Risk Prevention Models

Tyler Simmons Roger Williams University
Advisor(s): Yajni Warnapala, Roger Williams University

Multiple logistic regression allows several different components of an experiment to be observed in how they affect an overall outcome. Using the Behavioral Risk Factor Surveillance System survey data provided by the Rhode Island Department of Health, life satisfaction and preventative health models were constructed through a public health perspective. With these models it is possible to identify which aspects of peoples everyday lives have the greatest effect or impact on an individuals overall health and life satisfaction. To determine which variables had the largest impact, or how they compared to other variables, several different methods were used including the log odds and odds ratios. As these were predictive models, they can be used for future years to give the Rhode Island Department of Health recommendations for better public health outcomes. The statistical analysis used for this project was performed on JMP, the windows version of SAS.

324. Utilizing Multilevel Classification to Predict Adverse Drug Effects and Reactions

Tori Puhl Butler University
Advisor(s): Rasitha Jayasekare, Butler University

Multi-class classification models are used to predict categorical response variables with more than two possible outcomes. A collection of multi-class classification techniques such as Multinomial Logistic Regression, Support Vector Machines, Nave Bayes, and k-Nearest Neighbors is used in predicting patients drug reactions and adverse drug effects based on patients demographic and drug administration. The newly released, 2018 data on drug reactions and adverse drug effects from U.S. Food and Drug Administration are tested with the models. The applicability of model evaluation measures such as Kolmogorov-Smirnov, sensitivity, specificity and discordant ratios in multi-class settings, are also discussed.

325. Predicting Student Performance and Engagement Using Weighted Least Squares and Resistant Regression

Abigale Wynn Butler University
Advisor(s): Rasitha Jayasekare, Butler University

When using classical regression models, it is assumed that the error variance is constant for all the values of the predictor variables, resulting in homoscedasticity. When the assumption of the homoscedasticity fails (aka heteroscedasticity)

and presence of the outliers affects the regression model, the method of Weighted Least Squares and Resistant Regression provide an alternative for a better fit of the model. The goal of the project is to apply the method of Weighted Least Squares and Resistant Regression techniques to predict the student performance and engagement in college, using the data from National Survey of Student Engagement. The validity of the models and accuracy of predictions are also discussed.

326. Determining Effective Statistical Techniques to Analyze Cryptographic Systems

Jennefer Maldonado Adelphi University
Advisor(s): Salvatore Giunta, Adelphi University

The demonstration of complexities in encryption algorithms are vital when trying to encrypt important messages that hold information we wish to hide from adversaries. However, looking behind the statistics of different forms of encryption can be quite intriguing. In recent years, researchers have been studying the reliability of p-values and whether they are a trustworthy form of determining if data sets are statistically significant. To further analyze these data sets we will use Receiver Operating Characteristics (ROC) and Bayesian testing methods. We will take these advanced statistical methods of determining the level of significance of data sets and compare them with traditional null hypothesis statistical tests to demonstrate the effectiveness of these better methods.

327. Probabilistic Counting-Out Game on a Line

Tingting Ou Johns Hopkins University
Michelle Shu Johns Hopkins University
Advisor(s): John Wierman, Johns Hopkins University

Our research focuses on a novel problem posted on a question-and-answer website. There are n people in a line at positions $1, 2, \ldots, n$. For each round, we randomly select a person at position k, where k is odd, to leave the line, and shift the people at position i such that $i > k$ to position $i - 1$. We continue to select people until there is only one person left, who then becomes the winner. We are interested in two questions: which initial position has the greatest chance to win and which has the longest expected time to stay in the line. We have derived recursions to solve for exact values of the winning probabilities and expected time, the exact formula for the winning probabilities, and the asymptotic behaviors of the expected survival time. We have also considered a variation of the problem, where people are grouped into triples, quadruples, etc., and the first person in each group is at the risk of being selected. We will also present various sequences we have discovered while solving the variations, as well as other possible extensions and related findings concerning this problem.

328. Comparing University Rankings: Statistical Analysis of Four Global University Ranking Systems

Weiru Chen University of Illinois at Urbana-Champaign
Qianqian Li University of Illinois at Urbana-Champaign
Advisor(s): A J Hildebrand, University of Illinois at Urbana-Champaign

University rankings are provided by companies and organizations such as US News and World Report, Shanghai Ranking, Quacquarelli Symmonds (QS), and Times Higher Education. These rankings play a major role in a university's ability to recruit top students and faculty and to attract government funding and resources. In our research we seek to answer questions such as the following: How similar are the rankings produced by the different ranking systems? How similar are subject-specific rankings to overall rankings? Are there significant regional differences in rankings produced by different providers? To investigate these questions, we analyzed data from the four ranking systems mentioned above using well-known ranking metrics such as Kendall tau and the Spearman footrule measure, as well as statistical tests such as the paired comparison test and the Wilcoxon rank-sum test. We found,for example, that Shanghai Ranking and US News and World Report produced the most similar results overall, and that the discrepancies in rankings produced by the different ranking systems are significantly greater for universities in Asia and Europe than for North American universities.

329. Cluster Analysis of Drugs and their Adverse Effects

Brittney Man Butler University
Advisor(s): Rasitha Jayasekare, Butler University

Cluster analysis is an unsupervised technique that groups observations together based on similarities and dissimilarities to one another. Clustering can lead to uncovering meaningful patterns among the data. The goal of this project is to cluster drug information, patient reaction, and patient outcome data from the FDAs Adverse Events Reporting System (FAERS). Utilizing the measures such as Gap Statistics, Silhouette Coefficient and Dunn Statistics, the optimal number of clusters and how well the data are clustered will be determined. The clusters created from statistical methods such as K-Means and Agglomerative Hierarchical Clustering will then be compared and analyzed for nontrivial patterns or specific variables that may drive the clustering to better understand the relationships among the FAERS data.

330. Analyzing Voter Behavior in the Lehigh Valley Through Semi-Parametric Regression and Geostatistical Techniques

Benjamin Lieberman Muhlenberg College
Advisor(s): James Russell, Muhlenberg College

Predicting voter behavior, and voter trends has been a missing piece that has flummoxed mathematicians, and politicians alike. In order to predict voter tendencies, semi-parametric basis splines and logistic regression with spatial information was used. A variogram modeling technique was considered for modeling spatial dependence of voters within the Lehigh Valley, however no significant spatial dependence was apparent. This method relied on spatial data to analyze voter behavior, and explain voter tendencies and trends. Some significant predictors for voting turnout were: Political party, age, gender and location.

331. A Statistical Analysis of Muhlenberg College's Fourth Down Strategy

Luke Wiley Muhlenberg College
Advisor(s): James Russell, Muhlenberg College

This project aims to mathematically suggest whether a coach should go for it on 4th down by using generalized linear regression models to help probabilistically make the decision. We use the recent offensive data from Muhlenbergs football team. Most of the data is from the 2017 season; however, we also use previous years to supplement the kicking data. The data consists of decisions made on 3rd and 4th down play. Since 4th down usually means suggests a change of possession, 3rd down becomes critical of this decision. Benefits to Division III football include providing an edge over their competitors. The impact of 4th down decisions were recently on display in the past Super Bowl LII where the Eagles elected to employ a more aggressive, nuanced 4th down strategy while the Patriots followed a more conservative, traditional strategy.

332. A Model For Sequential Processes That Allow for Temporary Setbacks Before Terminating.

Zach Hollis Trine University
Dylan Kunce Trine University
Advisor(s): Daniel Dobbs, Trine University

An excited random walk is a random walk in which transition probabilities depend on the number of times a prior state has been visited. We use an excited random walk with loops to model sequential processes that allow any given state to be revisited a finite number of times before continuing onto the next state. We describe a Markov chain model related to this excited random walk. This model can be applied to events like playing baseball, playing platform video games, and completing a sequence of courses.

333. Prediction-Based Statistical Models for Real Estate

Anna-Maria Berezovski Saint Joseph's University
Advisor(s): Hongjun Ha, Saint Joseph's University

This research investigates the many relationships that can be found within the housing market of real estate and how useful these relationships can be to predict prices of houses. Several statistical models based on regression analysis are created and tested to model the interdependence between input variables and a target variable in the context of machine

learning. In particular, we run intensive numerical k-fold cross validations to avoid over-fitting and detect significant variables for appropriate predictions. By using government data sources, we can utilize several different variables that appeal to different concerns in the purchasing of a residential real estate.

334. Application of Matrix Methods in Financial Risk Analysis

Phillip Pender Montclair State University
Advisor(s): Aihua Li, Montclair State University

Financial risk analysts use historical time series data that details financial market factors to analyze stock prices, to measure general risk factors, and to build financial portfolios. We use daily stock closing prices as our data for creating correlations. The correlation matrix must be positive definite. When a correlation matrix is not positive-definite, we perform a spectral decomposition of the original correlation matrix and then undertake a nearest matrix search to find a nearest correlation matrix which is positive-definite. We also test the distance between the adjusted matrices and original correlation using various norms in order to determine which positive definite matrix is closest to the original correlation matrix. Python is used for data collection, numerical calculations, and testing the positive-definiteness of the output matrices. We also find the time complexity of the algorithms in order to see how viable this method is for larger data sets.

335. Quantitative and Local Central Limit Theorems

Annie Chen Stanford University
Ben Heller Stanford University
Eyob Tsegaye Stanford University
Advisor(s): George Schaeffer, Stanford University

We prove a quantitative local limit theorem for the number of descents in a random permutation. Our proof uses a conditioning argument and is based on bounding the characteristic function $\phi(t)$ of the number of descents. We also establish a central limit theorem for the number of 3-term arithmetic progressions (3-APs) in a random subset of $\mathbb{Z}/n\mathbb{Z}$. We conjecture that there is no local limit theorem for 3-APs, but a proof of this remains elusive. A promising avenue of proof is to condition on the size of the subset and show that the resulting distributions are too far apart for different sizes. This has proven difficult because the distances between these conditioned distributions on are the same order as their standard deviations such that the constant multiple between them is not very large. This research was conducted as part of the 2018 Stanford Undergraduate Research Institute in mathematics.

336. Bayesian Probabilistic Change Point Analysis

Rui Qiang College of the Holy Cross
Advisor(s): Eric Ruggieri, College of the Holy Cross

In regression analysis, a change point is defined as the point as which the statistical properties of a model change. Once identified, the data between any two change points can be fit by a homogeneous model. However, the locations of change points are generally unknown and must be inferred from the data. There are a variety of approaches used in identifying change points in datasets, yet few of the approaches give uncertainty bounds on both the number and locations of change points. Using Bayes' Rule, we created a probabilistic model capable of inferring both the number and locations of change points. Bayesian approaches can be sensitive to prior assumptions, so a number of simulated data sets were created in order to test the performance of our model in terms of both the accuracy and precision of change point detection. Along the way, changes were made to the algorithm to further improve the accuracy of our model. We also looked at the effect of autocorrelation on the detection of change point locations and then applied our model to real datasets including the S&P 500 index, oil and gold prices.

337. Minimum Entropy Clustering of Functional Data

Erik Wendt Gettysburg College
Advisor(s): Kumer Das, Lamar University

An interesting problem in functional data analysis is that of clustering functional data. While we have many models which cluster discrete data, which can either lead to better analysis of data or be used as an analysis onto itself, it is much harder to cluster functional data. One of the difficulty with clustering functional data is the nonexistence of standard probability mass functions on the space of continuous functions, which makes it challenging to define a notion density of for functional data, among other things. In this project, we develop a minimum entropy clustering algorithm for functional data after defining cluster entropy for functional data. We then compare this algorithm with classical functional data clustering algorithms, like k-means clustering. This work was a part of the Lamar University REU program (Award Id 1757717).

338. Optimizing the Creditworthiness Threshold of a Bivariate Distribution

Victoria Knutson St. Olaf College
Advisor(s): Hui Gong, Valparaiso University

Financial institutions must evaluate credit applications when deciding to issue credit. Creditworthiness varies amongst the applicants. Creditors must decide which applications to accept in order to maximize profit. For this research, we assume applicants are divided into Good and Bad populations. We found optimal threshold values that maximized the creditor's profit under varying assumptions of Normal, Chi-Square, and Gamma distributions. To do so, we optimized the profit function with respect to the threshold value and we ran simulations to find the threshold value that maximizes the profit. This research was completed at the summer REU in Mathematics at Valparaiso University.

339. The financial value of knowing the distribution of stock prices in discrete market models

Ryan Craver University of Maryland
Skylyn Brock Colorado Mesa University
Advisor(s): Fabrice Baudoin, University of Connecticut

An explicit formula is derived for the value of weak information in a discrete time model that works for a wide range of utility functions including the logarithmic and power utility. We assume a complete market with a nite number of assets and a nite number of possible outcomes. Explicit calculations are performed for a binomial model with two assets. The case of trinomial models is also discussed. This is an REU project done during summer 2018 at the University of Connecticut

340. Analyzing The Relationship Between Demographic Characteristics and Math Attitudes of Elementary Students

Tracey Ramirez California State University, Monterey Bay
Advisor(s): Alana Unfried, California State University, Monterey Bay

Online surveys of student attitudes toward Science, Technology, Engineering and Math (STEM) were administered to 222 third-through-fifth grade students at five STEM-focused schools in the Monterey Peninsula Union School District (MPUSD) to evaluate the efficacy of the districts STEM education. This poster will discuss the creation of reproducible, auto-generated summary reports and statistical analysis of the survey data. The automated reports, generated using R, allow for reproducible data cleaning, de-identification, and visualization while data analyses examine the relationship between student demographics and their STEM attitudes. Preliminary findings using multiple linear regression analysis indicate that students primary language is a significant predictor of their mathematics attitudes when controlling on socioeconomic status and gender. In particular, students whose first language is neither Spanish nor English have significantly higher attitudes toward math (p=0.02) than English-speaking students. We will discuss additional findings, all which aid in identifying STEM education areas that need attention and support district leadership as they improve their STEM programs.

341. Statistical Analysis and Modeling of Exclusionary Discipline in K–12 California Public Schools

Skylyn Irby University of Mississippi
Nathalie Huerta California State University Channel Islands
Cristal Quiones Pomona College
Advisor(s): Joanna Navarro, University of California Los Angeles

Using statistical methods on data from California public schools, we conducted an investigation on the exclusionary discipline practices widely used throughout the nation today. Specifically, using multiple linear regression, odds

ratios, and machine learning, we built a classification model that predicts the number of students in a given district that will pursue higher education in University of California and California State University schools. Some variables we consider are race/ethnicity, community and school demographics, and the number of expulsions, suspensions, and dropouts in a given district. Previous studies show that removing students from the traditional learning environment through expulsion or suspension is negatively correlated with academic success. This study goes beyond finding a relationship by predicting a district's student success as defined by the number of graduates eligible to attend a University of California or California State University school. Additionally, we considered the positive correlation of exclusionary discipline and dropout rates.

342. Spatial Analysis of Risk Factors Affecting State Rates of Suicide in Young Americans

Emely Garcia Kean University

Advisor(s): Kathryn Cowles, University of Iowa

According to the Center for Disease Control, suicide was ranked the second leading cause of death in Americans ages 15 through 24 years in the United States. This project applies Generalized Linear Models (GLMs) and Spatial Generalized Linear Mixed Models (SGLMMs) to assess the relationships between several predictor variables and the suicide rates for young Americans in U.S. states. Statewide predictor variables considered included demographics, availability of mental health facilities, and gun control laws. We found that states with higher proportions of counties without mental health facilities tend to have a higher risk of death by suicide. Furthermore, Amerindians and Whites have a significantly higher rate of suicide, and even after controlling for income data and proportions of rural versus urban residents, the proportions of whites are positively associated with a higher suicide rate. In addition, certain categories of gun laws were found to be associated with a decreased rates of suicide.

343. Data Analysis of Wind Speed at LaGuardia Airport for Optimization of Wind Turbine Design

Sanjib Lamichhane City College of New York

Advisor(s): Malgorzata Marciniak, LaGuardia Community College

Data analysis is a process of inspecting, cleansing, transforming, and modeling data with a goal of discovering useful information, suggesting conclusions, and supporting decision-making. We performed data analysis of wind speed recorded by National Oceanic and Atmospheric Administration for 17 years at LaGuardia airport, which is 6.1 miles to LaGuardia Community College (LaGCC) where we aim to design and install our wind turbine. The purpose of this study was to discover if there exist significant trends in wind speed around LaGCC that would affect modeling of the wind turbine design. Using R Programming, we calculated shape and scale parameters of Weibull Probability Density Function (PDF). The results showed that the Rsquared values for shape and scale parameters were to be 0.3836 and 0.1349. The Rcritical (RCR) values being greater than R-squared values, the trends were not significant within the period of time. The result would be used to model a wind turbine that would optimize the Coefficient of Performance.

344. RNA sequenced data across studies to strengthen biomarker detection

Sarah Szvetecz James Madison University

Advisor(s): Nusrat Jahan, James Madison University

Various studies have identified the chicken embryo (Gallus gallus) as a useful model to study the retinogenesis process in humans. The goal of this project is to use data from two specific RNA sequence studies to investigate retina developmental biology. These studies are done in two different labs using different protocols, as such they cannot be compared directly. Study 1 contains chicken retina samples from day 3, 5 and 8; while study 2 has retina samples from day 8, 16, and 18 of embryonic developmental age. In this work, we identify an effective normalization method that allows us to combine these two studies after adjusting the differences in the sequence library depth. The results from the analysis performed after the data was combined, provides us with a set of target genes that play a key role in retina development.

345. Schur measures and their asymptotic behavior

Ahaan Rungta Massachusetts Institute of Technology (MIT)
Advisor(s): Vadim Gorin, MIT

The aim of our work is to show how the asymptotic behavior of the Poissonized Plancherel measure and certain more general distributions on Young diagrams can be analyzed. In the process, we will find the limit shape for the Plancheral random Young diagrams and study the point configuration and how it behaves near a bulk point. We will conclude with some examples using Schur processes and Markov chains.

346. Bayesian Approach to Red Sox Hitters

Joshua Clark The College of the Holy Cross
William MacDonald The College of the Holy Cross
Advisor(s): Eric Ruggieri, The College of the Holy Cross

Everyone loves to watch their favorite Red Sox player hit a home-run in Fenway Park. But what if we could predict how many home runs theyll hit in their next game? The goal of this project is to model some of the top hitters on the Red Sox in order to predict their hitting statistics in other stadiums. We take a Bayesian approach to modeling the underlying rate parameter for our Poisson model, which takes into account a number of different factors that go into each players ability to hit a home run. As an extension of our original model, we hope to construct a good model that can predict the number of home runs based on the dimensions of Fenway Park. We plan to apply this model toward predicting how well the Red Sox will perform in the World Series based off their season statistics as well as how they might perform next season.

347. Time to hamstring injury in soccer players

Danielle Sebring California State University, Fullerton
Advisor(s): Valerie Poynor, California State University, Fullerton

In sports analytics, researchers are interested in modeling the time to injury of players to establish injury prevention practices and improve player longevity. The mean residual life function offers additional insight to this setting. The mean residual life function describes the expected time to injury give the player is without injury at a particular time, t. If the function is increasing this implies the player is less likely to be injured as time goes, whereas decreasing implies the player is more likely to be injured over time. In our research, we consider a dataset that was simulated under a real-world scenario. The data set describes the time to a hamstring strain injury, or the last time in which the player was know to be injury-free, for 100 soccer players. A number of covariates are also available for each player. We apply several parametric survival regression models to these data and select our final model via k-folds cross validation. We identify covariates that are significant in predicting time to injury as well as provide inferential results for the mean residual life function.

348. Algebraic Curve Fitting in R

Philip Hossu Illinois Institute of Technology
Advisor(s): David Kahle, Baylor University

The application of techniques from computational algebraic geometry to problems in statistics brings new insight into problems regarding data analysis. We describe how the construct of a variety, or solution set to system of polynomial equations, can be applied to the problem of finding a best fit curve to a random set of points. Viewed from the proper perspective, the method can be seen as a generalization of the simple linear regression model taught in introductory statistics courses, with some important distinctions. We additionally present a code implementation in R to perform basic algebraic curve fitting and detail some experimental results.

349. Comparing Object Correlation Metrics for Effective Space Traffic Management

Julie Zhang University of Washington
Advisor(s): Minh Pham, UCLA

In the near future, the number of satellites in Low Earth Orbit (LEO) is expected to grow tenfold. Therefore it is important to determine optimal space traffic management systems under various conditions. One essential part of space traffic management is the problem of object correlation: Given an a priori distribution of each object in space at a

given time and noisy measurements of unknown objects at a later time, how can one best associate each measurement to an object? This process of correlation depends on the choice of metric to quantify the likelihood that a certain measurement pairs with a certain object. Many metrics are already defined and explored in the literature, such as Mahalanobis, Bhattacharyya, and Optimal Control Distance. We contributed to this discussion a flexible simulation framework for comparing the performance of these metrics in a variety of scenarios while varying many simulation inputs. We focused our analysis on the cases where satellites move towards a pinch point or move away from a pinch point, and a case that simulates a more realistic LEO-type environment. The data shows a preference for Mahalanobis overall. This work was done at UCLA IPAM.

350. Comparing the Transmuted Rayleigh and Transmuted Weibull Distributions

Jeffery Williams Marshall University
Advisor(s): Raid Al-Aqtash, Marshall University

In this project, we intend to compare two models. the transmuted Weibull distribution studied in 2011 by Aryal and Tsokos and the transmuted Rayleigh distribution, a submodel of the transmuted Weibull, studied in 2013 by Merovci. These models will be fitted to real data sets, and these fits will be compared. This comparison will involve three tests of parameter significance.

351. Forecasting Performance Through Analytics

Craig Peterson Dixie State University
Advisor(s): Vinodh Chellamuthu, Dixie State University

In recent years, analytics in the sports industry has become a crucial component in designing winning strategies. Several studies have been conducted to support the more traditional statistical methods such as the Pythagorean value, Effective Field Goal (EFG), Corsi, and Goals Versus Threshold (GVT) in their respective sports. These analytic techniques play a significant role in the player selection from the youth level to professional sports. By modifying these methods, we developed an extended model based on the work of Rob Vollman and Wayne L. Winston to show the importance of a players age and its significance in the draft. Furthermore, we focused our attention on hockey by analyzing NCAA and junior hockey career statistics of the top 2018 NHL draft picks by rating player performance to provide insights into each prospect. Using our extended model, we were able to project top NHL draft picks performances in their rookie season.

352. Quantum Singularities in Spherically Symmetric Black Hole Spacetimes

Drew Weninger United States Naval Academy
Advisor(s): Deborah Konkowski, United States Naval Academy

Classical general relativity predicts the existence of irremovable singularities, indicated by incomplete geodesic paths in spacetime. These singularities are prevalent in a host of relativistic spacetimes, including those of observable cosmological objects such as various black hole systems. However, by analyzing a quantum wave packet instead of geodesic incompleteness, the potential exists to "remove" or "heal" these singularities. In this case, no boundary conditions are needed to be put on the singularity. Our technique focuses on analysis of the spatial segment of the uncoupled, relativistic Klein Gordon wave operator for a massless scalar particle and determining if it is essentially self-adjoint. In particular, Weyl's limit point - limit circle criterion are used to determine self-adjointness. Through self-adjointness properties, the spacetime can be characterized as quantum mechanically singular or non-singular. In our study, timelike curvature singularities associated with a group of spherically symmetric spacetimes are analyzed. Our results indicate the wave operator is not essentially self-adjoint for these spacetimes. Hence, they contain quantum singularities.

353. On union-closed families hinged on optimization and integer programming

Lucien Dalton Umass Amherst
Drew Polakowski Umass Amherst
Bertram Thomas Umass Amherst
Brianna Amaral Umass Amherst
Advisor(s): Annie Raymond, Umass Amherst

A family of sets is said to be union-closed if the union of any two sets in the family is also in the family. The Union-Closed Family conjecture (also known as the Frankl conjecture) states: in a non-empty union-closed family, there exists at least one element which appears in at least half of the sets. This conjecture has been approached from various angles and remains unsolved today. We consider a new approach based on optimization and integer programming. With the help of these new tools, we present interesting computations and prove theorems related to the conjecture.

354. Coloring the Curve Complex with Rabbits, Corabbits, and Airplanes

Janet Huffman Indiana Wesleyan University
Advisor(s): Rebecca Winarski, Georgia Institute of Technology

A polynomial can be composed with a homeomorphism called a Dehn twist. The resulting map is topologically equivalent to a (potentially different) polynomial. Given a quadratic polynomial whose critical point is three-periodic under iteration, there are exactly three resulting polynomials (up to equivalence). These polynomials are called the rabbit, corabbit, and airplane and each of these polynomials has three marked points. For each simple closed curve in a plane with three marked points, we compose the rabbit by a Dehn twist about that curve. Using algebraic and topological methods, we classify the curves as the rabbit, corabbit, or airplane based on the equivalence of the composition. Research done at Georgia Institute of Technology.

355. Generalized Cell Decompositions of Nested Lorenz Links

William Coenen Wayne State College
Advisor(s): Rolland Trapp, California State University, San Bernardino

This research focuses on generalizing the cell decompositions of various nested Lorenz links to determine types of hyperbolic and octahedral links. By utilizing a class of generalized fully augmented links called nested links, we have classified types of nested Lorenz links to be hyperbolic based on their generalized cell decompositions. We have also classified subsets of these hyperbolic nested Lorenz links to be octahedral. The culmination of this research was creating and proving the Lorenz Hyperbolicity Theorem, which allows us to determine if a given left nested Lorenz link is hyperbolic based on the positioning of two specific strands in the link.

356. Equivariant Cut-Paste Operations on Manifolds

Ben Riley University of Kentucky
Advisor(s): Carmen Rovi, Indiana University

A Cut-paste operation on a smooth oriented manifold allows us to cut it along a codimension 1 submanifold, and then paste back using an orientation-preserving diffeomorphism of the resulting boundaries. This operation allows us to define an equivalence relation on n-dimensional manifolds, and form a group denoted SK_n. The SK groups have surprising connections to bordism, and Karras, Kreck, Neumann and Ossa studied a collection of exact sequences which allow us to understand how these groups differ from each other. A crucial role in the theory is played by another cut and paste group, SKK, which allows invariants on cut and paste equivalent manifolds to differ by an error term controlled by the glueing diffeomorphisms. There have been attempts in the literature to modify SKK groups into new versions which have a less anomalous behavior in dimension 4k+1. Nevertheless, we found that there is an error in the literature in the definition of these modified SKK groups, which we will discuss. Our main result is a new exact sequence relating the SK and SKK groups. We additionally generalize this sequence to an equivariant setting. This work was done as part of an REU at IU-Bloomington.

357. Using Order Parameters and Persistent Homology to Analyze Biological Aggregations

Lu Xian Macalester College
Advisor(s): Lori Ziegelmeier, Macalester College

In this project, we explore the dynamics of biological aggregations which are groups of organisms, such as fish schools, bird flocks, and insect swarms, formed through social interaction and coordinated behaviors like attraction, repulsion, and/or alignment. We aim to classify by parameter numerical simulations generated from the highly-cited Vicsek model using both topology and the classic alignment order parameter. The topology approach computes the persistent homology at all time values of a simulation and summarizes this information as a crocker plot. The order parameter approach computes the normalized average of the velocity (that is, the alignment), producing a time series of the simulation. The outputs of both approaches for every simulation are input as feature vectors to machine learning clustering algorithms. We show that clustering with topology yields better results than clustering with order parameter and therefore, topology can be used as a reasonable means for parameter identification.

358. A Class of Virtual Knots With Crossing Weight Zero

Griffen Potrykus University of Wisconsin - Milwaukee
Bethany Schueller University of Wisconsin - Milwaukee
Advisor(s): Carolyn Otto, University of Wisconsin - Eau Claire

While trying to distinguish if two or more (virtual) knots are the same, we generally invoke the use of certain invariant properties of these knots to help us achieve this goal. These invariants range from determinants to the answers of yes or no questions. In this poster, a particular virtual knot invariant known as the crossing weight number will be discussed, including a new theorem surrounding the properties of this invariant as it pertains to a certain class of virtual knot. The theorem demonstrates what happens to this invariant when we perform a specific satellite construction on a given virtual knot, the Whitehead doubles. Namely, that the weight of each crossing in a Whitehead double vanishes. This research was done as a part of the summer 2018 mathematics REU at the University of Wisconsin - Eau Claire.

359. Macular Degeneration Classification through Topology and Convolutional Neural Networks

Michael Marlett College of the Holy Cross
Advisor(s): David Damiano, College of the Holy Cross

Macular degeneration is a disease of the retina that affects the macula, the region in the center of the retina responsible for high-resolution, color vision that is possible in good light. The loss of central vision makes it difficult to recognize faces, drive, read, or perform other activities of daily life. The classification of macular degeneration in the retina involves the ability to differentiate between three different types of macular degeneration, drusen, choroidal neovascularization, and diabetic macular edema as well as healthy retina. In this project, we combine methods from computational topology and deep learning to analyze a data set consisting of 83,711 optical coherence tomography (OCT) images of healthy and diseased retinas. In the first stage of this project we have applied convolutional neural networks to this data set. Our goal is to improve upon non-topological based analyses of OCT images.

360. Trunk of Satellite and Companion Knots

Nithin Kavi none
Wendy Wu None
Advisor(s): Zhenkun Li, MIT

We study a knot invariant called trunk, and the relation of the trunk of a satellite knot with the trunk of its companion knot. Our first result is $t(K) \geq n \cdot t(J)$ where $t(\cdot)$ denotes the trunk of a knot, K is a satellite knot with companion J, and n is the winding number. To upgrade winding number to wrapping number, which we denote by m, the cost is a factor of λ where $0 < \lambda \leq 1$: $t(K) \geq \lambda \cdot m \cdot t(J)$. It is conjectured that $\lambda = 1$. Currently we have proved the above inequality for $\lambda = \frac{1}{2}$. To do so, we translate the problem into a question about arranging surfaces on a 2-sphere and prove the corresponding statement combinatorially. Using this method, we also construct a sequence of intermediate values $\mu(i)$ so that $\mu(1) = m$ and for any $i \geq 1$, we have $n \leq \mu(i+1) \leq \mu(i) \leq m$ with $t(K) > \frac{i}{i+1} \cdot \mu(i) \cdot t(J)$.

361. Volume of Torus Links

Maya Klaib University of Redlands
Advisor(s): Rolland Trapp, CSUSB

We determine that all left nested (p,q) torus links are octahedral. We generalize the number of octahedra that make up left nested and complete nested (p,q) torus links. We define the T-links with octahedral parents in order to study these links and give an upper bound for their left nesting and complete nesting. We give a lower bound for this class of octahedral links. This research was done as an REU program at California State University San Bernardino

362. Parity of Linking Number for Two-Bridge Links

Timothy Ablondi Centre College
Yifan Ye Centre College
Advisor(s): Prayat Poudel, Centre College

Two-bridge knots and links, also known as rational knots/links, are of fundamental importance in the study of DNA recombination. Any two-bridge knot/link can be associated to a unique rational number, p/q. In our poster, we will

show how the continued fraction decomposition of the fraction p/q is related to the two-bridge link, and prove the relationship between p/q and the parity of its linking number.

363. Systole Length and Preservation Under Belt-Sums of the Borromean Rings

Amanda Cowell University of Michigan - Dearborn
Advisor(s): Rolland Trapp, California State University, San Bernardino

Through the use of Möbius transformations, the gluing maps are established for the fundamental polyhedron of the complement of the Borromean rings Ω. Then, these transformations are used to describe curves with the base point x_0 belonging to the fundamental group of Ω, denoted $\pi_1(\Omega, x_0)$. Taking the quotient space (reducing the set of curves to the equivalence classes), the curves which have systolic length of $\ell = 2.12255$ are located. From here, relevant thrice-punctured spheres S_i are identified and the existence of a systole living in $\Omega \setminus S$ for each of case of these thrice-punctured spheres is determined. Finally, it is uncovered why a particular type of manifold lacking a curve of length ℓ must not include the Borromean rings belt-summand. This work was completed under the guidance of Rolland Trapp during during the 2018 REU program at California State University, San Bernardino, and was funded under the NSF grand number 1758020.

364. Generating Set for all Nonzero Determinant Links Under the Skein Relation

Aayush Karan MIT PRIMES, University School of Milwaukee
Advisor(s): Jianfeng Lin, Massachusetts Institute of Technology

All links can be assigned a characteristic quantity known as the link determinant, obtained by plugging -1 into the Alexander Polynomial. This can be defined equivalently by deleting a row and column from the coloring matrix and taking the absolute value of the determinant of the resulting matrix. We consider a set of links $\mathcal{S}$ with the following two conditions: a) The unknot is contained in $\mathcal{S}$ b) Suppose three links with nonzero determinant constitute a skein triangle. Then if any two of these belong to $\mathcal{S}$, so does the third. By examining the behavior of the link determinant under the skein relation through a powerful method of decomposing links, we prove that for unoriented skein triangles, $\mathcal{S}$ is the set of all nonzero determinant links. Furthermore, if we allow the links to be oriented, we show that S only additionally needs the orientations of the Hopf Link as generators to contain all nonzero determinant links. This answers a question raised by Mullins in 1993.

365. Links on the Real Projective Plane

Isaac Weiss The College of Wooster
Advisor(s): Robert Kelvey, The College of Wooster

In this presentation, we discuss the work we conducted this summer as part of The College of Wooster's Applied Methods and Research Experience (AMRE). We do so by exploring a convention for drawing links on the real projective plane ($\mathbb{R}P^2$). We provide an algorithm that guides the viewer to build their own links on $\mathbb{R}P^2$. We find braid words for some links on $\mathbb{R}P^2$ and end with a series of theorems equating certain $\mathbb{R}P^2$ links.

366. Topological data analysis: a spectral approach

Amanda Merkley Colorado State University
Advisor(s): Patrick Shipman, Colorado State University

Persistent homology (PH) is a technique of geometric-topological data analysis applied to discrete data sets. PH encodes the dependence of topological information (such as number of connected components and number of holes) on a connectivity parameter. This parameter effectively gives the scale at which data points are connected to form a simplex, from which combinatorial methods are used to compute topological characteristics of the data. Traditionally, this parameter is based on a distance function on the data. We propose a spectral variation on PH that makes use of the Fourier transform of the distance function to compute the topological data. We demonstrate that spectral PH can be used as a substitute to current methods for obtaining certain topological information (connected components). We also show how spectral PH highlights dense clusters of data points (high frequency) and global topology (low frequency) by leveraging the spectrum of the distance function. This work was done as part of an REU at Colorado State University.

367. Topological Structure of Reaction Diffusion System

DeAndre Johnson Virginia State University
Advisor(s): Junping Shi, The College of William and Mary

In this project, we use concepts from algebraic topology to study the spatiotemporal dynamics of a predator-prey system. The spatiotemporal behavior of the system is described by reaction-diffusion equations. We find the numerical solution of the system using Finite Difference Method where we discretize the spatial dimensions to a 400 by 400 mesh grid to find the concentration value of each population. Complicated patterns are formed from the spatial dispersion of the population concentration at each time step. We then quantify the pattern by betti numbers from cubical homology. We observe the asymptotic behavior of betti number time series that greatly reduces the dimension of quantifying the pattern over time.

368. $SL(2,Z)$ Action on Quadratic Differentials

Paige Helms University of California, Riverside
Advisor(s): Tarik Aougab, Brown University

Title: $SL_2(\mathbb{Z})$ action on Quadratic Differentials The purpose of this research is to establish a lower bound for the number of orbits of the $SL_2\mathbb{Z}$ action on the space of Quadratic Differentials, cotangent bundle to Teichmüller space. We accomplish this through an algebraic interpretation of a pair of minimally intersecting curves that fill a surface $\Sigma_{g,0}$ of genus g with no boundary components or punctures, where for $g \geq 2$, the minimal intersection number is $i(\alpha, \beta) = 2g - 1$. Such a pair of curves can be visualized on a square-tiling of a surface $\mathbb{S}_g$ that also carries the structure of an origami. This interpretation gives us a way to examine the action of $SL_2\mathbb{Z}$ on a given surface and calculate its monodromy group, giving us a lower bound for the number of orbits.
We have shown the existence of one orbit in $n = 5$, four orbits in $n = 7$, three orbits in $n = 9$, and at least two orbits in $n = 11$.
Research conducted at ICERM's 2018 REU in conjunction with Brown University, funded through the NSF.

369. Algebraic k-Systems of Curves

Max Lahn Brown University
Simran Nayak Brown University
Aisha Mechery Bryn Mawr College
Advisor(s): Jonah Gaster, McGill University

For a positive integer k, a collection Δ of curves on an orientable surface S, possibly with boundary, is an algebraic k-system if each pair of distinct curves in Δ has geometric intersection number and (unsigned) algebraic intersection number k. We prove upper bounds on certain collections of vectors in finite-dimensional symplectic modules over $Z/2^{(m+1)}Z$, and deduce upper bounds on the size of an algebraic k-system on a compact, orientable surface. By constructing examples of large 2-systems on all closed surfaces and large k-systems on closed surfaces of small genus, we show that our bounds are almost tight infinitely often. This research was conducted at the Summer@ICERM REU hosted by Brown University.

370. Gordian Adjacency for Positive Braid Knots

Sam Serra University of Colorado Boulder
Luke Seaton Louisiana Tech University
Advisor(s): Katherine Raoux, Michigan State University

A knot K_1 is said to be *Gordian adjacent* to a knot K_2 if K_1 is an intermediate knot on an unknotting sequence of K_2 (Feller 2014). Determining Gordian adjacency requires knowing the unknotting numbers in advance, so we choose to study Gordian adjacency between positive braid knots, particularly torus knots. Our main approach is to develop algorithmic methods for determining adjacencies via manipulating the associated braid words. Doing so, we obtain new adjacencies between torus knots as well as certain other positive braid knots. We also use this approach to show there exists only finitely many positive braid knots of a given unknotting number. This research was conducted as part of the SURIEM REU program at Michigan State University under the direction of Dr. Robert Bell and mentorship of Dr. Katherine Raoux.

371. Automorphisms of the k-curve graph

Yassin Chandran UCSB
Roberta Shapiro Rutgers University
John (Rob) Oakley Concordia University Texas
Advisor(s): Tarik Aougab, Brown University

The k-curve graph of an orientable surface S with negative Euler characteristic is a graph whose vertices correspond to (homotopy classes of) essential simple closed curves on S, and whose edges correspond to pairs of curves that geometrically intersect at most k times. For any surface with genus at least 3, we prove that the automorphism group of the 1-curve graph is isomorphic to the extended mapping class group; this resolves a conjecture of Schaller from 2000. More generally, we prove the same result for the k-curve graph so long as the absolute value of the Euler characteristic of S is at least $512+k$. This work was done during Summer@ICERM 2018: Low Dimensional Topology and Geometry.

372. Monotonicity of Length of Closed Geodesics Given Self-Intersection Number

Isnayni Hadi California State University, Northridge
Brandis Whitfield Brandeis University
Alex Xu University of California, Santa Barbara
Advisor(s): Moira Chas, Stony Brook University

Fix a hyperbolic surface S, and for each $k \geq 0$, define l_k(S) as the length of the shortest closed geodesic in S with exactly k self-intersection points. We wish to prove that $l_k(S)$ is increasing; i.e. the shortest closed geodesic with $k+1$ self-intersections is longer than the shortest closed geodesic with k self-intersections. Previous work has shown that $\lim_{k\to\infty} l_k(S) = \infty$, yet it is still an open problem whether this sequence is strictly increasing. We study surgery on closed curves to understand how it affects the self-intersection number for any given curve on a hyperbolic surface and use this to obtain partial results to this problem. Our work towards this problem was produced through the Summer@ ICERM 2018: Low Dimensional Topology and Geometry program.

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