

Probability and Stochastic Processes Course Area

Chair: Jean Johnson, *Baker University*.

Committee Members: Saeed Ghahramani, *Western New England University*; Matt Richey, *St. Olaf College*; Mike O'Neill, *Claremont McKenna College*, Moustapha Pemy, *Towson University*

Introduction. Probability plays a critical role in the theory and methods of a wide range of mathematical sciences and is an important field in pure mathematics in its own right. Variability, randomness, risk, and related attributes characterize most measurable phenomena in the real world. Probability has a central place in the mathematics curriculum and provides a foundation for the study of mathematical statistics, stochastic optimization, actuarial studies, stochastic physical models, financial mathematics, and more.

Prerequisites. This report concerns the courses in probability intended for the mathematics major as well as for other mathematically-intensive fields. We assume that at least two semesters of calculus are prerequisite to the first course; many schools also require multivariate calculus and linear algebra. Although there are worthwhile probability courses that do not require a calculus background, we do not discuss them here.

Essential Considerations

- **Elegance and practicality.** Probability theory is both mathematically elegant and remarkably practical. As with continuous and discrete mathematics, the distinction between deterministic and stochastic modeling is fundamental to mathematics. Yet not until 1933 did Andrei Kolmogorov place probability theory on a firm theoretical foundation. The theoretical results that followed had implications for applications, where there are many subtleties about how to interpret probabilities in real world settings. The resolution of many subtleties rests on the axioms, definitions, and theorems in a carefully constructed probability course. Probability courses should be based on the set of commonly agreed-upon axioms, and be developed by providing precise definitions and theorems. The proofs in a first introductory course are generally accessible to undergraduates and should be derived as a standard part of the course.

Scientists and engineers have long relied on probability theory: physicists (quantum mechanics, Brownian motion, and statistical physics), chemists (dynamics of chemical reactions), biologists (classical genetics and evolutionary biology), geologists (flow through porous media and earthquake prediction), and engineers (telecommunications, quality control). Probability theory is also becoming more important in business, formerly mainly in insurance but now in finance as well. For a recent example consider the problem of how to properly model events that are

perceived as regularly recurring but rare, such as a stock market crash or a 100-year flood. The course should reveal the practical nature of the theory through presentation of a myriad of word problems and a variety of applications. It is important to build courses that reveal both the rigor and the practicality of the subject.

- **Relationship to statistics.** Probability theory forms the basis of mathematical statistics. Much of classical statistics depends crucially on probability theory; even data-driven statistics can be approached with a Bayesian view, itself mathematically defensible and dependent on probability. Much of operations research deals with modeling and optimizing stochastic systems such as queuing and inventories. Calculus-based probability courses are often constrained by the requirements of courses that follow, such as Statistics, Stochastic Processes, and Operations Research. This can present a dilemma in choosing which of the many topics to present in the first post-calculus course.
- **Preparation of teachers.** Many mathematics departments prepare future mathematics teachers. Especially in light of the increasing importance for American society of data and the understanding of the nature of risk, departments should help future teachers understand the basic ideas in probability, data analysis, and statistics. In particular, the Common Core State Standards in Mathematics require that teachers be prepared to introduce some of the mathematics of random phenomena. This certainly implies that pre-service mathematics teachers should complete a statistics course. In addition, pre-service teachers of secondary school mathematics would benefit from a calculus-based probability course. These courses show students how many of the topics from calculus are applied in the treatment of both discrete and continuous probability distributions, and in both univariate and multivariate distributions. This includes the use of an axiomatic system to build the theory, the summation of a variety of infinite series, the applications of techniques of integration, and so on.
- **Technology.** The subject of probability, though rooted in an axiomatic foundation and supported by a robust theoretical structure, is increasingly enriched by the power of computing. Pedagogically, students now have the opportunity to more easily comprehend some of the complex, and often surprising, results of probability theory.

Examples abound: Let X , Y , and Z be independent random variables, uniform on $[0, 1]$. What is the probability that the roots of the quadratic $Ax^2 + Bx + C$ are real? (Source: Rice third edition, exercise 3.11). The answer to this question is $(5 + \sqrt{64})/2 \approx 0.254413$, and involves either a double or triple integral (depending on how one tackles it). However, the code to simulate this situation, written in R,

estimates the probability to several decimal places. The code is short (five lines) and runs almost instantaneously. Even the famous (infamous?) "Monty Hall" problem can be easily and transparently simulated. Bayesian statistics, an increasingly important area of probability, is inherently computational, as are Markov chains.

Exposing students of probability to computation goes beyond the substantial pedagogical value. Probability as it is practiced today, either in an academic or industrial setting, is often heavily dependent on computations to support and inform conclusions. Consider, for example, computer network analysis, models based on large data sets, and actuarial analysis. Whatever a probability student's potential future career, she or he will be better served by having been exposed to many ways in which probability theory and computing intersect.

Given the rapidly changing nature of both hardware and software, we are reluctant to make recommendations regarding specific software for use in class. R, an open source product, is increasingly popular because of the ease in which probability distributions and simulations are handled. Computer algebra systems such as Maple, Mathematica, and Matlab also have robust packages to support probabilistic work. Graphing calculators and Excel, despite their ubiquity in our lives, are not recommended as tools for use in a modern probability course.

- **Recognizing and Coping with Demands.** Departments with concentrations in actuarial science or in statistics may have little control over the content and pace of the first probability course. Students who intend to take the first few actuarial examinations while still undergraduates will want to be prepared for the scope and depth of the exam. Departments with many pre-service secondary mathematics teachers may choose to emphasize combinatorics, assuming that there will be a separate (and perhaps non-calculus based) statistics course with emphasis on data analysis. If designed primarily for pre-service teachers, probability courses may contain more in the way of simulation and experimentation. Small departments often have to make a choice. The pace of a probability course meant as a prerequisite for actuarial science or for a mathematical statistics course may not allow the kind of exploration and experimentation that would be best for pre-service teachers. Many varieties and approaches are possible. We discuss several options below.
- **Mathematical and Cognitive Goals.** Probability courses teach students, above all, how to think systematically about uncertainty and randomness. Students learn that rigorous, logical reasoning is needed to read, model and analyze probability problems. They experience the use of a simple axiomatic system in working with probability defined on properly constructed sample spaces. They learn

combinatorial reasoning in working with discrete sample spaces. The course mixes discrete and continuous mathematics. Although a model may be discrete, methods for summing infinite series become essential. Students learn the important idea that the sum of discrete random variables approaches, as n grows, a limiting continuous distribution which is often easier to work with than the original distribution—infinity can be your friend! Another key concept in probability is conditioning—how partial knowledge changes how one thinks about a problem.

A probability course builds students' skill in looking for multiple strategies for solving a problem. It encourages better understanding of the role of modeling in applied mathematics, and leads to some degree of comfort in incorporating the role of risk when modeling real-life applications. Students should have the opportunity to use technology to clarify basic ideas and help them form conjectures. Students must understand definitions and theorems thoroughly and precisely before they can use them. They get practice in producing examples and counterexamples and in creating simple proofs. Students should be responsible for producing proofs and cogent arguments in any version of this course. A calculus-based probability course shows students important applications of the ideas of calculus.

Probability courses offer excellent opportunities for presenting strategically designed exercises that can enhance students' cognitive development. Students begin to understand the role of an axiomatic system, see the need for precise definitions, and appreciate the utility of set theory and proof in the structure of the subject. They should be encouraged to create their own proofs and to defend their solutions.

We particularly recommend that students be encouraged to recognize that they may have to master an entire (and somewhat extensive) tool kit in order to solve many problems. Students are exposed to the uses of calculus and the power of the limiting process as they tackle applications and learn strategies in problem solving. There may be many proofs of the same theorem, referring back to the axioms, using set theory, generating functions, recursion, or using a combinatorial way of thinking. Using a variety of approaches enhances and extends the student's basic understanding of and ability to handle complex processes.

The use of simulation and manipulatives gives more concrete evidence of the need to check results against a theoretical structure. The history of probability contains many missteps of even the most famous mathematicians. Presentation of supposed paradoxes and dilemmas can be a powerful learning experience for our students!!

We strongly encourage students to discuss their solutions because the variety of worthwhile approaches to a problem is itself instructive. It encourages two-way

communication – listening and speaking interactively with one’s peers. This is particularly true in solving combinatorics problems and serves as a powerful message to future teachers about the necessity of seeing a problem from a variety of vantage points. Reconciling different answers to the same problem often reveals unwarranted assumptions or faulty reasoning. These are good exercises for all mathematics students.

Overview of Different Probability Courses

Many departments offer more than one *upper division* course in (calculus-based) probability, although small departments may be able to offer only one. At this time, the two most common probability offerings are a full one-semester course in probability and a probability-statistics course in which probability is the major component. *This study group recommends that departments move towards offering a full semester course in probability and that most students majoring in mathematics should take such a course.* Due to changes in statistics education, this Curriculum Guide assumes that in the future, mathematics majors and most other STEM students will take a lower-division data analysis course (with some classical statistics). This assumption may obviate the need for integrating statistics into a first probability course.

We recognize that some departments may need to continue to offer an upper-division probability-statistics course for client groups such as computer science or engineering majors whose programs do not allow for separate courses in data analysis, statistics, and probability. The standard syllabus for such courses has been stable for many decades and appears to meet the needs of these client groups. We list some appropriate texts in the bibliography.

Valuable upper-division courses in probability may have several possible foci: one oriented towards stochastic processes and mathematical finance, one towards combinatorial methods, and another towards mathematical statistics. We leave discussion of the combinatorics course for the course area report on Discrete Mathematics.

As mentioned above, the applied data analysis course is assumed to be an additional course, and we leave the discussion of the contents of that course for the Statistics Area course report.

Upper-division Probability course oriented towards mathematical statistics. This course is often offered as the first semester of a two-semester sequence in probability and mathematical statistics. Its prerequisite is calculus up through multivariable integration. A

course like this has existed for almost 100 years and is oriented towards follow-up study in classical statistics. It was long the sole beginning probability course available for actuarial studies, although the newer stochastic processes-oriented probability course is now deemed acceptable by the Society of Actuaries.

The course starts with basics about sample spaces, axioms of probability, random variables and their distributions (discrete as well as continuous), probability density and distribution functions, expectation and variance. Sometimes this course includes several weeks devoted to combinatorial enumeration. Bivariate and multivariate situations lead to consideration of conditional probability and independence for discrete and continuous random variables. At this point, the statistics-oriented course generally diverges from the stochastic processes-oriented course.

The statistics-oriented course next spends considerable time with joint and multivariable distributions. This includes extensive technical calculus calculations of marginal density functions followed by functions of two or more random variables, with more technical work with transformations of variables and Jacobians. The material involves more calculus than probability, but the work is essential for further calculations of the distributions of important statistics of random samples.

Moment generating functions and their properties are used to find expectations and variances of common discrete and continuous distributions along with the distributions of their sums. The idea of a generating function is a new concept to most students, but the power of the concept is a strong message to students. Generating functions may be introduced at the beginning of the course and then used to compute expectations and variances when common discrete and continuous random variables are first discussed. The course usually ends with the study of limiting distributions, including the Laws of Large Numbers and the Central Limit Theorem, giving significant attention to limiting processes.

Overall, this course has a technical focus in laying the foundation for future work in mathematical statistics. The most widely used text for this course is the first half of *Probability and Statistical Inference*, by Robert Hogg and Elliott Tanis, 8th ed., Prentice-Hall, though our bibliography contains many stand-alone probability texts that work as least as well.

Upper-division Probability Course oriented toward stochastic processes. This course has the same multivariable integration prerequisite as the course oriented toward mathematical statistics and the syllabus is the same up through the common discrete and continuous distributions, as noted above. Applied moment generating functions may be introduced. However, the course goes more deeply into conditional probability and independence and into diverse applications of the discrete and continuous distributions. Overall, the course has a greater emphasis on using probability concepts and formulas for

diverse problem solving. The technical, calculus-based skills of the other course get less attention.

This course devotes limited time to multivariate and joint distributions except for sums of independent identically distributed random variables. Here too, conditional distributions get extra attention. Expectations are treated in great depth including conditional expectations. Chebyshev's inequality, the Laws of Large Numbers, and the Central Limit Theorem usually finish the course. Sometimes this course also covers some Markov chains.

The widely used text for this course is *A First Course in Probability* by Sheldon Ross, 8th ed., Prentice-Hall. Other popular texts are listed in the bibliography.

Stochastic Processes or a second Probability course. A second undergraduate probability course, if offered at all, is usually about stochastic processes. Stochastic processes are important in operations research, applied mathematics, computer science, actuarial science, engineering, and financial mathematics. Either version of the probability course can be a prerequisite, although obviously the stochastic process-oriented version is better.

Stochastic processes can be approached in an applied fashion as the stochastic side of operations research, or with a more mathematically-oriented approach. Standard topics include Poisson processes and renewal theory, queuing theory, and discrete and continuous Markov chains. The operations research version would include extensive applications to inventory theory. The mathematical version would also treat Brownian motion.

In this course, students should formulate models, gain analytical skills, perform and analyze simulations, and present results to others. They should be encouraged to use computational tools where appropriate and to follow more involved proofs than in the prior (required) probability course. For more about a stochastic operations research course, see the Operations Research Program Area report.

Textbooks

Probability:

- *Probability: The Science of Uncertainty*, Michael A. Bean, American Mathematical Society, 1st Edition, 2009.

This book covers the basic probability of distributions with an emphasis on applications from the areas of investments, insurance, and engineering. Written by a Fellow of the Casualty Actuarial Society and the Society of Actuaries with many years of experience as a university professor and industry practitioner, the book is suitable as a text for senior

undergraduate and beginning graduate students in mathematics, statistics, actuarial science, finance, or engineering as well as a reference for practitioners in these fields.

- *Probability: With applications and R*, Robert P. Dobrow, Wiley, 1st Edition, 2013.
- *Elementary Probability for Applications*, Rick Durrett, Cambridge University Press, 1st Edition, 2009.
This concise introduction to probability concentrates on the results that are the most useful for applications, including combinatorial probability and Markov Chains. It is designed for a one-semester introductory course in probability for students who have some familiarity with basic calculus. Reflecting the author's philosophy that the best way to learn probability is to see it in action, the examples contain the standards, but also include a number of applications from areas as broad ranging as genetics, sports, finance, and inventory management.
- *The Essentials of Probability*, Richard Durrett, The Duxbury Press, 1st Edition, 1994. This book was designed for a senior-level probability course at Cornell University. It describes the theoretical results briefly and then illustrates them by a number of examples. It has only 5 chapters, but more than enough material for one course.
- *Probability and its Applications*, Martin M. Eisen and Carole A. Eisen, Quantum Pub, 1st Edition, 1975.
This is a high school algebra-based textbook, very well written for students with little mathematical background. The book is out of print, but still is used at some institutions. It can be purchased from used book distributors.
- *Fundamentals of Probability with Stochastic Processes*, Saeed Ghahramani, 3rd Edition, Pearson, 2004.
This book presents probability in a natural way; it uses interesting, carefully selected instructive examples that explain the theory, definitions, theorems, and methodology. It provides some brief history about probability and most important, concrete process of the proof of theorems, many of which are attractive to students.
- *Probability and Random Processes*, Geoffrey R. Grimmett and David R. Stirzaker, Oxford University Press, 3rd Edition, 2001.
This book gives an introduction to probability and its many practical application by providing a thorough, entertaining account of basic probability and important random processes, covering a range of important topics. Emphasis is on modelling rather than abstraction and there are sections on sampling and Markov chain, Monte Carlo, renewal, reward, queueing networks, stochastic calculus, and option pricing in the Black-Scholes model for financial markets. In addition, there are almost 400 exercises and problems relevant to the material. Solutions can be found in *One Thousand Exercises in Probability*.

- *Probability: An Introduction*, Geoffrey Grimmett and Dominic Welsh, Oxford Science Publications, 1st Edition, 1986, Reprinted 2009.
This undergraduate text offers a concise introduction to probability and random processes. Exercises and problems range from simple to difficult, and the overall treatment, though elementary, includes rigorous mathematical arguments. Chapters contain core material for a beginning course in probability, a treatment of joint distributions leading to accounts of moment-generating functions, the law of large numbers and the central limit theorem, and basic random processes.
- *Introduction to Probability*, Charles M. Grinstead and J. Laurie Snell, 2nd Revised Edition, American Mathematical Society, 1997.
This book presents a thorough treatment of ideas and techniques necessary for a firm understanding of probability. The text is also recommended for use in discrete probability courses. The material is organized so that the discrete and continuous probability discussions are presented in a separate, but parallel, manner. This organization does not emphasize an overly rigorous or formal view of probability and therefore offers some strong pedagogical value. Hence, the discrete discussions can sometimes serve to motivate the more abstract continuous probability discussions.
- *Introduction to Probability Theory with Contemporary Applications*, Lester L. Helms, Dover Publications Inc., 1st Edition, 2010.
The book exposes a student to diverse applications of probability through numerous examples, convincing the student that there is coherent set of rules for dealing with probabilities and there are powerful methodologies for solving probability problems.
- *Introduction to Probability Theory*, Paul G. Hole, Sidney C. Port, Charles Stone, 1st Edition, Cengage Learning, 1972.
The strongest feature of this book is its conciseness. Much is presented in as short a time as possible. The basic explanations are clear with many instructive examples. In addition to conciseness, the authors have made a commendable effort to present the reader with clear and concrete definitions and compact theorems (many proven).
- *Introduction to Probability and Its Applications*, Richard M. Larsen and Morris L. Marx, 1st Edition, Pearson, 1985.
Drawing heavily on real-world examples and case studies, this book offers a calculus-based, non-measure theoretic, problem-solving-oriented introduction to probability.
- *Probability Models and Applications*, Ingram Olkin, Leon J. Gleser, Cyrus Derman, Prentice Hall College Div., 2nd Edition, 1994.

This text promotes cross-disciplinary research into the modeling of the ever-increasingly complex data involved in scientific and technological research. It also provides a unique emphasis on where and how to apply probability models to real phenomena and an unusually strong preparation in the tools necessary for such applications.

- *Modern Probability Theory and its Applications*, Emanuel Parzen, Wiley Interscience, 1st Edition, 1992.

This text introduces probability theory, showing how probability problems can be formulated mathematically to systematically attack routine methods. Topics include independence and dependence, probability laws and random variables. Over 500 exercises, an appendix of useful tables and answers to odd-numbered questions are also included.

- *Probability*, Jim Pitman, 1st Edition, Springer Verlag, 1999.

The emphasis of this book is to understand how to apply probability to some real-world problems. It includes excellent, intuitive examples which are non-trivial and easily extended. It also includes many examples which give both an intuitive/thinking-about-it solution and a formulaic solution.

- *A First Course in Probability*, Sheldon Ross, 9th Edition, Pearson, 2012.

The writing in this book is very clear but concise; there are plenty of examples and exercises for each chapter. The three sets of exercises in the back of each chapter are problems, theoretical exercises, and self-test problems and exercises. Ross gives selected answers to exercises and shows some fully worked out.

- *Introduction to Probability*, George Roussas, Academic Press, 1st Edition, 2006. This text features explanations of the mathematics of probability theory and explores its diverse applications through numerous motivational examples. It provides a thorough introduction to the subject for advanced undergraduate students taking their first course in probability. The content is based on the introductory chapters of Roussas's book, *An Introduction to Probability and Statistical Inference*, with additional chapters and revisions.

- *A Course in Probability*, Neil A. Weiss, Pearson, 1st Edition, 2005.

This text is intended primarily for readers interested in mathematical probability as applied to mathematics, statistics, operations research, engineering, and computer science. It is also appropriate for mathematically oriented readers in the physical and social sciences. Prerequisite material consists of basic set theory and a firm foundation in elementary calculus, including infinite series, partial differentiation, and multiple integration. Some exposure to rudimentary linear algebra (e.g., matrices and determinants) is also desirable.

Probability and Statistics:

- *Essentials of Mathematical Statistics*, Brian Albright, Jones & Bartlett Learning International Series in Mathematics, 1st Edition, 2013.
This book is written for the one-term introductory probability and statistics course for mid-to upper-level math and science majors, *Essentials of Mathematical Statistics* combines the topics generally found in mainstream elementary statistics books with the essentials of the underlying theory. The book begins with an axiomatic treatment of probability followed by chapters on discrete and continuous random variables and their associated distributions. It then introduces basic statistical concepts including summarizing data and interval parameter estimation, stressing the connection between probability and statistics. Final chapters introduce hypothesis testing, regression, and non-parametric techniques. Many sections include a sub-section titled “Software Calculations” which gives detailed descriptions of how to perform the calculations discussed in the section using the software Minitab, R, Excel, and the TI-83/84 calculators.

- *Probability and Statistics with Applications : A Problem Solving Text*, Leonard A. Asimov and Mark M. Maxwell, 1st Edition, ACTEX Publication, Inc., 2010.
A Problem Solving Text is an introductory textbook designed to make the subject accessible to college freshmen and sophomores concurrent with their study of calculus. It is organized specifically to meet the needs of students who are preparing for the Society of Actuaries and Casualty Actuarial Society qualifying examination P/1. Sample actuarial exam problems are integrated throughout the text along with an abundance of illustrative examples and 799 exercises. Additionally, the chapters on mathematical statistics cover all of the learning objectives for the statistics portion of the Casualty Actuarial Society Exam 3L syllabus. Liberal use is made of past exam problems from CAS Exams 3 and 3L. The book provides the content to serve as the primary text for a standard two-semester advanced undergraduate course in mathematical probability and statistics.

- *Probability and Statistical Inference*, Robert Bartoszynski and Magdalena Niewiadomska-Bugaj, 2nd edition, Wiley-Interscience, 2008.
This book is intended as an upper-level undergraduate textbook, but can be used to teach lower-level graduate students as well. It treats probability and statistics as self-contained subjects, with their own conceptual structures and methods taken from mathematics. The material is presented in a way that students are forced to spend time and effort on reflection, even at the cost of sacrificing some of the pragmatic values.

- *Introduction to Probability*, Dimitri P. Bertsekas and John N. Tsitsiklis, Athena Scientific, 2nd Edition, 2008.

The book is the currently used textbook for an introductory probability course at the Massachusetts Institute of Technology, attended by a large number of undergraduate and graduate students. The book covers the fundamentals of probability theory (probabilistic models, discrete and continuous random variables, multiple random variables, and limit theorems), which are typically part of a first course on the subject, as well as the fundamental concepts and methods of statistical inference, both Bayesian and classical. It also contains a number of more advanced topics, from which an instructor can choose to match the goals of a particular course. These topics include transforms, sums of random variables, a fairly detailed introduction to Bernoulli, Poisson, and Markov processes. Some of the more mathematically rigorous analysis has been just intuitively explained in the text, but is developed in detail (at the level of advanced calculus) in the numerous solved theoretical problems.

- *Statistical Inference*, George Casella and Roger Berger, 2nd Edition, Cengage Learning, 2001. This book builds theoretical statistics from the first principles of probability theory. Starting from the basics of probability, the authors develop the theory of statistical inference using techniques, definitions, and concepts that are statistical and are natural extensions and consequences of previous concepts. Intended for first-year graduate students, and also suitable for strong undergraduate honors students, this book can be used for students majoring in statistics who have a solid mathematics background. It can also be used in a way that stresses the more practical uses of statistical theory, being more concerned with understanding basic statistical concepts and deriving reasonable statistical procedures for a variety of situations, and less concerned with formal optimality investigations.
- *Probability and Statistics*, Morris H. DeGroot and Mark J. Schervish, 4th Edition, Pearson, 2011. This text presents a balanced approach of the classical and Bayesian methods and includes a chapter on simulation (including Markov chain Monte Carlo and the Bootstrap), coverage of residual analysis in linear models, and many examples using real data. Calculus is assumed as a prerequisite, and a familiarity with the concepts and elementary properties of vectors and matrices is a plus. The book introduces concepts with a verbal description of the goal and the underlying intuition before delving into the mathematics.
- *Modern Introduction to Probability and A Statistics: Understanding Why and How*, F.M. Dekking, C. Kraaikamp, H.P. Lopuhaä, and L.E. Meester, Springer Verlag, 1st Edition, 2007. This book shows how the fundamentals of probabilistic and statistical theories arise intuitively. It provides a tried and tested, self-contained course, that can also be used for self-study. It has numerous quick exercises to give direct feedback to the students. In addition the book contains over 350 exercises, half of which have answers, of which half have full solutions. The website www.springeronline.com/1-85233-896-2 gives access to the data files used in the text, and, for instructors, the remaining solutions. The only prerequisite for the book is a first course in calculus; the text covers standard statistics and

probability material, and develops beyond traditional parametric models to the Poisson process, and on to useful modern methods such as the bootstrap.

- *Probability and Statistics: The Science of Uncertainty*, Michael J. Evans and Jeffrey S. Rosenthal, W. H. Freeman; 2nd Edition, 2009.
This book brings a flavor based on incorporating the computer to the course and an integrated approach to inference. From the start the book integrates simulations into its theoretical coverage, and emphasizes the use of computer-powered computation throughout. Students with just one year of calculus can use this text and experience a blend of applications and theory that goes beyond merely mastering the technicalities. An integrated approach to inference is presented that includes the frequency approach as well as Bayesian methodology. Bayesian inference is developed as a logical extension of likelihood methods. A separate chapter is devoted to the important topic of model checking and this is applied in the context of the standard applied statistical techniques. Examples of data analyses using real-world data are presented throughout the text. A final chapter introduces a number of the most important stochastic process models using elementary methods. An appendix in the book contains Minitab code for more involved computations. The code can be used by students as templates for their own calculations. If a software package like Minitab is used with the course then no programming is required by the students.
- *Introduction to Mathematical Statistics*, Robert V. Hogg, Joseph McKean, and Allen T. Craig, 7th Edition, Pearson, 2012.
This classic book retains its outstanding ongoing features and continues to provide readers with excellent background material necessary for a successful understanding of mathematical statistics. Chapter topics cover classical statistical inference procedures in estimation and testing, and an in-depth treatment of sufficiency and testing theory—including uniformly most powerful tests and likelihood ratios. Many illustrative examples and exercises enhance the presentation of material throughout the book. It is suitable for an honors course in an advanced undergraduate mathematical statistics course, or a beginning graduate course.
- *Probability and Statistical Inference*, Robert V. Hogg and Elliot Tanis, Pearson, 8th Edition, 2009.
This classic textbook, written by two leading statisticians, applies introduction to the mathematics of probability and statistics emphasizing the existence of variation in almost every process, and how the study of probability and statistics helps us understand this variation. Designed for students with a background in calculus, this book continues to reinforce basic mathematical concepts with numerous real-world examples and applications to illustrate the relevance of key concepts.

- *Introduction to Mathematical Statistics and Its Applications*, Richard M. Larsen and Morris L. Marx, 5th Edition, Pearson, 2011.
Noted for its integration of real-world data and case studies, this text offers sound coverage of the theoretical aspects of mathematical statistics. The authors demonstrate how and when to use statistical methods, while reinforcing the calculus that students have mastered in previous courses.
- *Mathematical Statistics and Data Analysis*, John A. Rice, Duxbury Advanced, 3rd Edition, 2006.
This book's approach interweaves traditional topics with data analysis and reflects the use of the computer with close ties to the practice of statistics. The author stresses analysis of data, examines real problems with real data, and motivates the theory. The book's descriptive statistics, graphical displays, and realistic applications stand in strong contrast to traditional texts that are set in abstract settings.
- *A Course in Mathematical Statistics*, George G. Roussas, 2nd edition, Academic Press, 1997.
This book contains enough material for a year-long course in probability and statistics for advanced undergraduate or first-year graduate students, or it can be used independently for a one-semester (or even one-quarter) course in probability alone. It bridges the gap between high and intermediate level texts so students without a sophisticated mathematical background can assimilate a fairly broad spectrum of the theorems and results from mathematical statistics. The coverage is extensive, and consists of probability and distribution theory, and statistical inference.
- *Mathematical Statistics, A Unified Introduction*, George R. Terrell, Springer Verlag, 1st Edition, 1999.
This textbook introduces the mathematical concepts and methods that underlie statistics. The course is unified, in the sense that no prior knowledge of probability theory is assumed, being developed as needed. The book is committed to both a high level of mathematical seriousness and to an intimate connection with application. It directly appeals to intuition and manages to provide some hunch for advanced topics. Moreover, it remains connected to empirical examples and encourages students to engage with the data.

Stochastic Processes

- *Basic Stochastic Processes* by Zdzislaw Brzezniak and Tomasz Zastawniak, Springer Undergraduate Mathematics Series, (2000).
This textbook provides a concise and self-contained introduction to the theory of stochastic processes. The idea of conditional expectation, which is the backbone of the theory of stochastic processes, is developed in considerable detail, which gives an excellent preparation for the study of martingales, Markov chains and Brownian motion in the

subsequent chapters. There are numerous exercises scattered all over the chapters with full solutions at chapter ends. With this book you are allowed to explore the rigorous side of stochastic calculus, yet maintain a physical insight of what is going on. The authors have concentrated on the most important and useful topics that are encountered in common physical and financial systems.

- *Introduction to Stochastic Processes* by Erhan Cinlar, Dove Publications (2013)
This book emphasizes the modern viewpoint, in which the primary concern is the behavior of sample paths. By employing matrix algebra and recursive methods, rather than transform methods, it provides techniques readily adaptable to computing with machines. Topics include probability spaces and random variables, expectations and independence, Bernoulli processes and sums of independent random variables, Poisson processes, Markov chains and processes, and renewal theory. Assuming some background in calculus but none in measure theory, the complete, detailed, and well-written treatment is suitable for engineering students in applied mathematics and operations research courses as well as those in a wide variety of other scientific fields.
- *Markov Processes for Stochastic Modeling* by O. Ibe, Academic Press (2008)
This textbook, which is written for upper level undergraduate and graduate students, and researchers, presents a unified presentation of Markov processes. In addition to traditional topics such as queuing system, the book discusses such topics as continuous-time random walk, correlated random walk, Brownian motion, diffusion processes. Continuous-time random walk is currently used to model the financial market, which has traditionally been modeled as a Brownian motion. Correlated random walk is popularly used in ecological studies to model animal and insect movement. Hidden Markov models are used in speech analysis and DNA sequence analysis while Markov random fields and Markov point processes are used in image analysis. Thus, the book is designed to have a very broad appeal.
- *A First Course in Stochastic Processes*, by Samuel Karlin and Howard M. Taylor, Academic Press (1975)
The book is written in a classical mathematical style, where the authors take you by hand through the difficult passages and derivations. The main intuition about stochastic processes is so well conveyed, and the mathematics so well explained, that the book can be read with little or no recourse to pencil and paper. The book presents a comprehensive foundation of the theory of stochastic processes. It is great textbook of senior and first year graduate courses.
- *An Introduction to Probability and Stochastic Processes* by James L. Melsa and Andrew P. Sage, Dove Publications (2013)
This text offers students valuable tools for the description and analysis of random effects in complex systems. The treatment emphasizes concepts with practical value and illustrates

their application to realistic situations. Geared toward college seniors and first-year graduate students, this volume is designed for a one-semester course in probability and stochastic processes. The sole prerequisite is a familiarity with system analysis, including state-variable and Laplace-transform concepts, and two appendixes provide a review of these concepts. Topics covered in detail include probability theory, random variables and their functions, stochastic processes, linear system response to stochastic processes, Gaussian and Markov processes, and stochastic differential equations. The coverage extends to major concepts essential to modern systems analysis and presents several applications of theory to realistic practical problems in science and technology. Numerous examples and problems appear throughout the text, many with worked-out solutions.

- *Probability, Statistics and Stochastic Processes* by Peter Olofsson, Wiley-Interscience (2007)
This textbook highlights the interrelationships between probability, statistics, and stochastic processes. Beginning with three chapters that develop probability theory and introduce the axioms of probability, random variables, and joint distributions, the book goes on to present limit theorems and simulation. The authors combine a rigorous, calculus-based development of theory with an intuitive approach that appeals to readers' sense of reason and logic. It is an excellent book for courses on probability and statistics at the upper-undergraduate level. The book is also an ideal resource for scientists and engineers in the fields of statistics, mathematics, industrial management, and engineering.

- *Stochastic Processes* by Sheldon M. Ross, Wiley Series in Probability and Statistics (1995)
This textbook provides a non-measure theoretic introduction to stochastic processes. Considers its diverse range of applications and provides readers with probabilistic intuition and insight in thinking about problems. This revised edition contains additional material on compound Poisson random variables including an identity which can be used to efficiently compute moments; a new chapter on Poisson approximations; and coverage of the mean time spent in transient states as well as examples relating to the Gibb's sampler, the Metropolis algorithm and mean cover time in star graphs. Numerous exercises and problems have been added throughout the text.

- *A First Course in Stochastic Models* by H. C. Tijms, Wiley & Sons (2007)
This textbook is a self-contained introduction to the theory and applications of stochastic models. Emphasis is placed on establishing the theoretical foundations of the subject, thereby providing a framework in which the applications can be understood. Without this solid basis in theory no applications can be solved. It is suitable for senior undergraduate and graduate students from computer science, engineering, statistics, operations research, and any other discipline where stochastic modeling takes place. It stands out amongst other textbooks on the subject because of its integrated presentation of theory, algorithms and applications.

