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There are many geometric properties that are easy to prove when you are in a particular case. For instance, it is trivial to show that the medians of an equilateral triangle or those of a right isosceles triangle are concurrent. But what if the special case were the general case? Any triangle can be sent by an affine transformation to a given equilateral or right isosceles triangle. The medians of the triangle are sent to the medians of the new triangle and concurrence of three lines is preserved by affine transformations. Hence, the theorem is proved in the general case.

This far-reaching idea is the theme of the article at hand. It is in the spirit of the Erlangen Program published by Felix Klein in 1872, in which different geometries are characterized by their groups of transformations and the corresponding invariants. The group of affine transformations is the natural group for Euclidean geometry. The paper explores this group at length. Since an affine transformation is completely determined by the image of three non-collinear points, all triangles are affine equivalent. Parallelism is preserved by affine transformations, and all parallelograms are affine equivalent. Trapezoids are affine equivalent if and only if their base ratios are equal, hence the family of trapezoids depends on a positive real parameter. When one considers a quadrilateral, any three consecutive vertices can be sent to$(1,0)$, $(0,0)$ and $(0,1)$, yielding two parameters $(a,b)$ for the position of the fourth vertex. These simple, but deep, considerations are used throughout the paper to elegantly prove several theorems of Euclidean geometry.

The last example discussed is that of Pascal's theorem, which states that if the vertices of a hexagon lie on an ellipse and the three pairs of lines containing the opposite sides intersect, then the three points of intersection are collinear. Using an affine transformation, it suffices to prove the theorem when the ellipse is a circle. This example is an opportunity for the authors to open a window on the extension to projective geometry. In this larger context, not only are all ellipses projectively equivalent to a circle, but also to all parabolas and hyperbolas. Moreover, in projective geometry, parallel lines intersect at infinity. Hence, the extension to projective geometry gives for free the result that if the vertices of a hexagon lie on a conic, then the points of intersection of the three pairs of lines containing the opposite sides are collinear.

This paper is truly a jewel. It presents very deep ideas, and it is likely to have a profound influence on anyone teaching Euclidean geometry. The writing is extremely clear and engaging, and the diagrams are very helpful. This paper should be very readable by students and provides a nice selection of exercises to keep all readers entertained.

### Response

We are deeply honored to be selected for the George Pólya Award. George Pólya, who outstandingly played the role of mathematician, educator, and writer, was a true intellectual of his time, and it is genuinely humbling to have his name within any neighborhood of ours. We all share a deep love of mathematics, and a passion for sharing it. We hope that this article will bring insight and new perspective to many readers, and we are overjoyed by the recognition of our efforts to advance the philosophy and accessibility of the Erlangen program. The College Mathematics Journal is a splendid and highly motivating editorial environment, inviting us all to better explain—at a fundamental level—the mathematical ideas that matter the most to us. We are also extremely grateful for the very inspiring intellectual atmosphere permeating the Department of Mathematics at CSU Fullerton. For being instrumental in creating this wonderful depart-

mental culture, we are particularly indebted to our retired department chair, Stephen W. Goode, forever our friend and colleague.

Adam Glesser, Matt Rathbun, and Bogdan Suceavă would like to thank their respective doctoral advisors, Robert Boltje, Abigail Thompson, and Bang-Yen Chen, from whose lectures, papers, and books they learned how to think and how to write. Isabel Serrano would like to thank the MAA and its members for introducing her to mathematics' versatility and power to provide insight into numerous fields. As an undergraduate who consistently attended regional and joint meetings, these gatherings exposed her to what a career in research could be and provided the network and motivation to continue her academic career.

## Biographical Sketch

**Adam Glesser** received his PhD in 2006 from the University of California, Santa Cruz, where he worked under the direction of Robert Boltje studying open conjectures in the representation theory of finite groups. He has since worked at the University of Aberdeen (Scotland), Suffolk University (Boston), and for the last eight years, California State University, Fullerton. His work has appeared in, among others, *Trans-actions of the American Mathematical Society, Journal of the London Mathematical Society*, as well as on his YouTube channel. He is a proud father of three boys with whom he loves spending time playing sports, board games, and music. He is also in awe of his superhero wife who, along with taking care of everybody at home, helps bring new life into the world as a labor & delivery nurse.

**Matt Rathbun** studied Mathematics at the University of California, Berkeley, and earned his Ph.D. at the University of California, Davis. After a year at Michigan State University, and three years at Imperial College London, he has been very fortunate to return to California and is finishing his sixth year at California State University, Fullerton. His research is in pure and applied knot theory and low-dimensional topology, though his student research projects and enthusiasm for teaching keep him quite busy with new and fascinating mathematics.

**Isabel Serrano** earned a BA in Mathematics with a Concentration in Applied and Computational Mathematics at California State University, Fullerton. In addition to her love of mathematics, Isabel pursued a minor in History and co-authored papers with Dr. Bodgan Suceavă focused on the history of mathematics. During her undergraduate career, Isabel also worked on mathematically modeling the 2016 Zika virus outbreak under the guidance of Dr. Anael Verdugo. Currently, Isabel is pursuing her PhD in Computational Biology at the Universi-ty of California, Berkeley, where she is employing computational approaches to study the diversity in aging.

**Bogdan D. Suceavă** graduated from the University of Bucharest, Romania, earning a BSc in Mathematics in 1994 and a MSc in Geometry in 1995. He pursued his doctoral program with Michigan State University and defended his doctoral thesis, written under Bang-Yen Chen's supervision, in 2002. His works have appeared in the *Houston Journal of Mathematics, Taiwanese Journal of Mathematics, American Mathematical Monthly*, the *Mathematical Intelligencer, Beiträge zur Algebra und Geometrie, Differential Geometry and Its Applications, Czechoslovak Mathematical Journal, Publicationes Mathematicae, Results in Mathematics, Notices of the American Mathematical Society,* and several other journals. Two of his papers (both co-authored with Isabel M. Serrano, and one co-authored with Lucy H. Odom) have been selected for the *Best Writings in Mathematics* anthologies, published by the Princeton University Press. Besides his mathematical papers, he is the author of 17 literary books written in Romanian, including the novels *Coming from an Off-Key Time* (Northwestern Univ Press, 2011) and *Miruna, a Tale* (Twisted Spoon Press, 2014), which was presented with the Bucharest Writers Associ-ation Fiction Award 2007.