

Paul R. Halmos - Lester R. Ford Awards

Jan E. Holly

“What Type of Apollonian Circle Packing Will Appear?,” *The American Mathematical Monthly*, 128:7, 611–629. doi.org/10.1080/00029890.2021.1933834

This visually striking article invites readers on an exploration of Apollonian circle packings, packings that are determined iteratively by three circles, or two circles and a line, that are pairwise tangent. A pair of ratios of radii of the circles (which is zero if one circle is a line) determines whether a packing fills a circle, a strip, a half-plane, or the full plane. The article focuses on the question, how can one tell which type of packing is determined by a particular ratio pair? Partial answers emerge from construction and analysis of a plot that reveals the general structure of the parameter space $[0,1] \times (0,1]$ of ratio pairs: open ovals and half-ovals of ratio pairs for packings of circles, boundary curves corresponding to half-plane and strip packings, and a fractal limit of those curves that comprises the ratio pairs for full-plane packings. At a more detailed level, the boundary curves are broken into a union of algebraic curves whose endpoints correspond to strip packings and each of whose interior points corresponds to a half-plane packing when not also an endpoint of another curve. The article stands out for its rich set of gifts: several directions for further inquiry and numerous specific questions for which the reader has been well-prepared by the author. Indeed, the author’s use of easily accessible analytic techniques, complex geometry, and visual reasoning, including definition of a key sub-packing, offer a reader both inspiration and multiple tools for further study.

Response

I am grateful to be honored, and am particularly excited about the prospect of additional results that readers may generate upon seeing the article. In working on this problem of types of Apollonian circle packings, it was a surprise how uncharted the territory is. As a consequence, I really wanted to share the fun with other people. Even students can tackle some of the open problems and/or explore the ideas creatively. This project began quite by accident. As an entertaining diversion from my research projects, I planned to jot down a few equations to figure out how to determine what type of Apollonian circle packing would appear. Surely the answer was already known and published! However, after a ridiculous number of equations and graphs, it became clear that this was going to be a project. Not only had no solution been published, but the solution involved an intriguing fractal with new questions arising at every turn. The *Monthly* is a fabulous publication, perfectly fitting my desire to share the excitement. Besides being accessible to a wide audience, The *Monthly* embraces mathematicians’ enjoyment not only of results, but also of new problems and challenges. Many thanks to the editors for facilitating the publication of my article, and for all of their valuable work over the years.

Biographical Sketch

Jan Holly recently completed 25 years as a professor at Colby College in Maine, with research especially focusing on human spatial disorientation in association with organizations such as NASA and the NIH. Meanwhile, pure mathematics has always been in the mix, beginning naturally in childhood trying to prove the then-unsolved Fermat’s last theorem (and, yes, solving the Rubik’s Cube from scratch), eventually followed by undergraduate work at the University of Colorado and the University of New Mexico, graduate work at the University of Illinois with PhD specializing in logic and algebra, a postdoctoral position at the Robert S. Dow Neurological Sciences Institute in Portland, Oregon, and then Colby College plus sabbatical stints at additional institutions. Jan is now a research associate affiliated with Colby College, residing in the state of Washington while exploring mathematics and mountains.